Mathematics G4403. Modern Geometry **Spring 2010**

Assignment 13

Due on Monday, February 1, 2010

- In this assignment, all the manifolds are *connected*.
- [dC]= do Carmo, *Riemannian Geometry*.

(1) Let $\mathcal{H}^n = \{(y_1, \ldots, y_n) \in \mathbb{R}^n \mid y_n > 0\}$ be the n-dimensional upper half space. For any $\alpha \in \mathbb{R}$, define a Riemannian metric g_α on $\mathcal{H}^n$ by

$$g_\alpha = y_n^\alpha (dy_1^2 + \cdots + dy_n^2).$$

By [dC] Chapter 8 Section 3, $(\mathcal{H}^n, g_{-2})$ is a complete Riemannian manifold. Prove that the Riemannian manifold $(\mathcal{H}^n, g_\alpha)$ is complete if and only if $\alpha = -2$. (Hint: consider the geodesic $\gamma(t)$ with $\gamma(0) = (0, \ldots, 0, 1)$ and $\gamma'(0) = \frac{\partial}{\partial y_n}$.)

(2) [dC] page 153 Exercise 8.

(3) [dC] page 154 Exercise 12.

(4) [dC] page 154 Exercise 13.