October 4,2024													
Introduction.													
A History of Algebraic and Differential Topology, 1900 - 1960	by Jee	an Dien	ndonn	í .									
REU pager by Christopher Stith													
countertion of TT, (S')													
covering spaces of (X, x0)													
(\mathbf{x}, \mathbf{x})													
0. History													
16th and 19th centuries													
Poincaré (1854 - 1912)													
prchistony of algebraic topology													
1873 = Kiemann and Betti													
Browner's work on appreximation													
Weigh's treatment of the theory of hickness curtaria													
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clements of T and open													
Tu Tu													
1.2: closed points													
1.3: connected space													
continuity: IR, IR ^R , topology													
1.4: §: X→Y is consimuous													
1.5: continuity for a path													
1.61 path-connected space													
" local properties , "toomed in" , "small"													
1.7% neighborhood of 94													
1.8: local path-connectedness													
Circle = square													
homeomorphism.													
1.91. fog = 14y. b. gos = 14y.													
1.10: homeomorphic													
homosopy equivalence													
" continuous deformation"													
homeomorphic => homotopy equivalent		• •											
1.11: H:X+I+Y.s.t. H(x,0)=f(x) and H(x,1)	= 9(2)	Vre	Χ.										
homotopic: 5≃g													
$h_{\ell}: X \to Y$													
he:= H X+ξεξ													
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I.14: homotopy equivalent w.r.t. f.og ≈ id, and g.	oş.≃ id _× .																	
1.16: contractible																		
1.17: null-homotopic																		
helo)=f(0)=g(0) and helis=f(1)=g(1) Afel																		
1.18: homotopy relative																		
2. The Fundamental Group																		
topological invariants																		
fundamental group																		
"(group at leap.")																		
2.1: loop in X is a path f with $f(0) = f(1)$																		
2.2; constant loop in X build at 2:0	t c L																	
2.4: path multiplication = f.g = 2 g(2t-1) +	5551																	
2.5! f''(t) = f(1-t)																		
. \$\$ 5 1200																		
is loop multiplication associative?																		
(§+9)·h ≃ 5·(9·h)																		
$[\frac{1}{2} \cdot \frac{1}{2}] = [\frac{1}{2} \cdot \frac{1}{2}]$																		
[\$]. [4] = [5]																		
[\$].[\$"]=[4]																		
2.6: fundamental group. The (X, x)																		
2.7: homomorphism $P_*([5]) = [po5]$								•	•									
2.8: Let X be a space and let $x_0, x_1 \in X$ in the sam	e path-con	nected	compo.	nent.	Thur	π,(X	ر قدر	≊.π,	(.x.,	را د								
Prost follows in lecture																		
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2.9: If X is para-connected, then $\forall \gamma_0, \gamma_1 \in X$, π ,	(x, x,).	≅π,(×,*,`). W	: then	retu	vnam	bigu	onely	o+.	Hhis	isen	racbj	mison	clou	N. 64	" me	
2.9: If X is para-connected, then $\forall x_0, x_1 \in X$, π , fundamental group of X_1^{n} , denoted $\pi_1(X)$	(א, אי) . יייי	≌π,(X, ~,`), Wi 	then	reter		bigw	بالمحمد) 1 0	this	`\\@ ^		nis en	clou	si. 64	18 Words 	
 2.9: If X is para-connected, then. ∀X₀, x, € X, π, fundamental group of X," denoted π₁(X) 2.10: simply-connected space 	(X, x,).	≌π,(X, ~,` 	. . .	then	refu		big w	onely	of.	*****	` ```		nis m	c).	n ene .	restine	
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(4) Let x E S'. VKEZ,	∃loop h: I → S' boned at x s	.t. n(h)= K		
Proof of 1,4 follows in lecture				
3.7: ₶,(\$') ≌.(ℤ,+)				
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HI' A country spaces	······································			
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$p^{-1}(V) = \bigcup_{i=1}^{N} \bigcup_{i$	1) are distining any sets in \tilde{X} s.	t_{\cdot} of \cdot : $V_{\cdot} \rightarrow V_{\cdot}$ is a home	morabian	
• p: X→x				
p ⁻¹ (x) ⇒ fibur				
degree n				
n-fold covering				
evenly covered				
'sheets				
eine, ei(x-s, x+s)				
ં H.2: p:X→X, 5:V→X,	š: Y→X., po5=5 · · ·			
4.3: homotopy lifting property				
. 4.4: based map , f: (X, xa) → (Y,y₀)			
4.5: covering space, injective	, ••• · · · · · · · · · · · · · · · · ·			
4.6.: lifting criterian				
4.7! Unique lifting property.				
4.7! Unique lifting property				
4.7: unique lifting property. isomorphism 1-10-1 correspondence. 4 R: 2 4 5				
4.7: unique lifting property. isomorphism. 1-10-1 correspondence. 4.8: $\tilde{X}_1 \stackrel{\Phi}{\to} \tilde{X}_1$				
4.7: unique lifting property isomorphism. 1-10-1 correspondence H.8: X, → X, P, X, P2				
4.7: unique lifting property isomorphism. 1-to-1 correspondence. H.&: X, → X, P, X, P;				
4.7: unique lifting property isomorphism. 1-10-1 correspondence 4.8: X, → X, P, X Pi 5. Covering Spaces and the Fi	indemental, Group			
4.7: unique lifting property isomorphism. 1-10-1 correspondence 4.8: $\tilde{X}_1 \stackrel{\Phi}{\longrightarrow} \tilde{X}_1$ $P_1 \stackrel{\Phi}{\longrightarrow} \tilde{X}_2$ 5. Covering Spaces and the Fo	indomental Group.			
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