



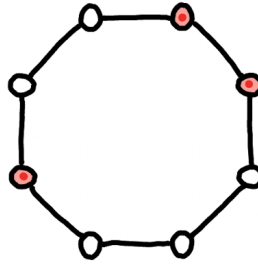
## Exam 2

Combinatorics, Dave Bayer, April 6-10, 2023

Please show all of your work. You will be graded for both your answers and your explanations. You need not complete the entire exam; some questions are intended to be challenging.

This test is open-book. You may use any resource such as my course materials, textbooks, or *The On-Line Encyclopedia of integer Sequences*. You may not receive help from another person.

[1] How many ways can we choose three vertices of an octagon, up to rotation?



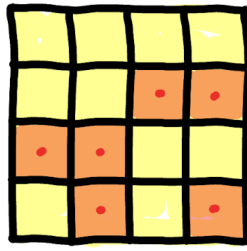


[2] Which dissection of a polygon corresponds to this Young tableau, under Stanley's correspondence?

1	2	6	7
3	5	8	10
4			
9			

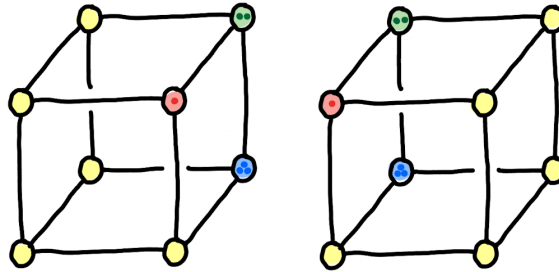


[3] let  $f(k)$  count the number of ways of coloring the squares of a  $4 \times 4$  grid using at most  $k$  colors, up to the dihedral group  $D_4$  of rotations and reflections of the square. What is  $f(2)$ ? What can you say about  $f(k)$ ?





[4] Color the vertices of a cube using at most  $k$  colors, up to rotations of the cube. Let  $f(k)$  count the number of *chiral pairs*: Mirror images that are not the same under rotation. What is  $f(4)$ ? What can you say about  $f(k)$ ?





[5] Let  $f(p)$  count the number of ways of coloring a  $p$  bead necklace using at most 3 interchangeable colors, up to rotation. In other words, we're partitioning the beads into up to 3 unnamed subsets, up to rotation. As shown,  $f(2) = 2$  and  $f(3) = 3$ . What is  $f(5)$ ? What can you say about  $f(p)$ , when  $p$  is prime?

