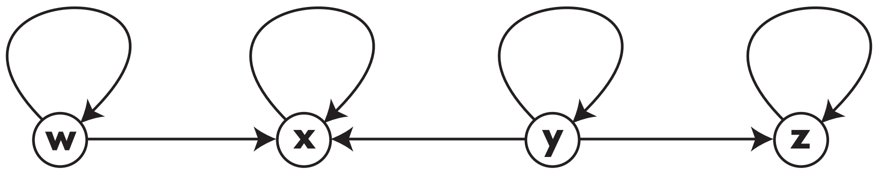
S14 Final Exam Problem 1
Linear Algebra, Dave Bayer

[1] Using matrix multiplication, count the number of paths of length ten from y to z.



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10 paths

S14 Final Exam Problem 2
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[2] Find the 3×3 matrix that projects orthogonally onto the plane

$$x - z \;=\; 0$$

$$\frac{1}{2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

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[3] Find $f(n)$, where $f(n)$ is the determinant of the $n \times n$ matrix in the sequence

$$\begin{bmatrix} 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 2 & 0 \\ 1 & 3 & 2 \\ 0 & 1 & 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 2 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 2 & 0 & 0 & 0 \\ 1 & 3 & 2 & 0 & 0 \\ 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

Your final answer should be in the form $\quad f(n) = a\, r^n + b\, s^n$

$$f(n) = 2 \cdot 2^n - 1$$

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[4] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix}$$

$$\lambda = 0, 2, 4 \qquad e^{At} = \frac{1}{4}\begin{bmatrix} 2 & 1 & -2 \\ 0 & 0 & 0 \\ -2 & -1 & 2 \end{bmatrix} + \frac{e^{2t}}{2}\begin{bmatrix} 0 & -2 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix} + \frac{e^{4t}}{4}\begin{bmatrix} 2 & 3 & 2 \\ 0 & 0 & 0 \\ 2 & 3 & 2 \end{bmatrix}$$

S14 Final Exam Problem 5
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[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$\lambda = 3, 1, 1 \qquad e^{At} = \frac{e^{3t}}{2} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & 2 & 0 \end{bmatrix} + \frac{e^{t}}{2} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ -2 & -2 & 2 \end{bmatrix} + te^{t} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

S14 Final Exam Problem 6
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[6] Solve the differential equation $y' = Ay$ where

$$A \;=\; \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \qquad y(0) \;=\; \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda = 0, 2, 2 \qquad e^{At} \;=\; \frac{1}{2}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} + \frac{e^{2t}}{2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix} + te^{2t}\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$y \;=\; \frac{1}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \frac{e^{2t}}{2}\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} + te^{2t}\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

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[7] Solve the differential equation $y' = Ay$ where

$$A \;=\; \begin{bmatrix} -2 & 2 & -1 \\ -1 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}, \qquad y(0) \;=\; \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda = 0, 0, 0$$

$$e^{At} \;=\; \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \;+\; t \begin{bmatrix} -2 & 2 & -1 \\ -1 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \;+\; \frac{t^2}{2} \begin{bmatrix} 3 & -3 & -3 \\ 3 & -3 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$y \;=\; \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \;+\; t \begin{bmatrix} -5 \\ -4 \\ -1 \end{bmatrix} \;+\; \frac{t^2}{2} \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$$

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[8] Express the quadratic form

$$3x^2 \;+\; 2xy \;+\; 2y^2 \;+\; 2yz \;+\; 3z^2$$

as a sum of squares of othogonal linear forms.

$$\lambda = 1, 3, 4 \qquad A \;=\; \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix} \;=\; \frac{1}{6}\begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix} + \frac{3}{2}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} + \frac{4}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\frac{1}{6}\,(x \;-\; 2y \;+\; z)^2 \;+\; \frac{3}{2}\,(x \;-\; z)^2 \;+\; \frac{4}{3}\,(x \;+\; y \;+\; z)^2$$