Final Exam

Linear Algebra, Dave Bayer, Section 003, May 14, 2013

Name: ______________________________ **Uni:** ______________

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	**Total**

If you need more that one page for a problem, clearly indicate on each page where to look next for your work.

[1] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

[2] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which projects orthogonally onto the line

$$x = y = z$$

Find the matrix A which represents L in standard coordinates.

[3] Let V be the vector space of all polynomials $f(x)$ of degree $\leqslant 2$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace of V consisting of those polynomials satisfying $f(0) = 0$. Find an orthogonal basis for W with respect to the inner product

$$\langle f, g \rangle \;=\; \int_0^1 f(x)g(x)\,dx$$

[4] Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & 2 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 4 & 1 \\ -4 & 0 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & -3 \\ -4 & 0 \end{bmatrix}$$

[7] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -1 & 1 & 1 \\ -2 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

[8] Express

$$x^2 + y^2 + z^2 - 2xy + 2xz + 2yz$$

as a linear combination of squares of orthogonal linear forms.