[6] Let A be the matrix

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}.$$

Find a basis of eigenvectors and eigenvalues for A. Find the matrix exponential e^A.

[7] Let A be the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

Find an orthogonal basis in which A is diagonal.

[8] Find a matrix A so the substitution

$$\begin{bmatrix} x \\ y \end{bmatrix} = A \begin{bmatrix} s \\ t \end{bmatrix}$$

transforms the quadratic form $x^2 + 4xy + y^2$ into the quadratic form $s^2 - t^2$.

[6] Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Find the eigenvalues and eigenvectors of A.

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answer:

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work:

Continued on back of page _____

[7] Let $A = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$. Find the matrix exponential e^{At}.

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answer:

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work:

Continued on back of page _____

[5] Let $A = \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 0 \\ 1 & 1 & -2 \end{bmatrix}$. Find the eigenvalues and eigenvectors of A.

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answer:

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work:

Continued on back of page _____

[6] Let $A = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$. Find the matrix exponential e^{At}.

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answer:

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work:

Continued on back of page _____

[7] Let $A = \begin{bmatrix} 0 & 1 & -1 \\ -2 & 3 & -1 \\ -2 & 2 & 0 \end{bmatrix}$. Find the matrix exponential e^{At}.

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answer:

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work:

Continued on back of page _____

[4] Let $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. Write A as CDC^{-1} for a diagonal matrix D.

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answer:

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work:

Continued on back of page _____

[5] Let $A = \begin{bmatrix} 2 & 1 & -2 \\ 2 & 1 & -2 \\ 3 & 1 & -3 \end{bmatrix}$. Write A as CDC^{-1} for a diagonal matrix D.

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answer:

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work:

Continued on back of page _____

[6] Let $A = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix}$. Find the matrix exponential e^{At}.

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answer:

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work:

Continued on back of page _____

[7] Let $A = \begin{bmatrix} 2 & 1 & -1 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Find the matrix exponential e^{At}.

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answer:

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work:

Continued on back of page _____

[4] Let $A = \begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}$. Write A as CDC^{-1} for a diagonal matrix D.

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answer:

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work:

Continued on back of page _____

[5] Let $A = \begin{bmatrix} -1 & 2 & -2 \\ -1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Write A as CDC^{-1} for a diagonal matrix D.

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answer:

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work:

Continued on back of page _____

[6] Let $A = \begin{bmatrix} 3 & 4 \\ 1 & 3 \end{bmatrix}$. Find the matrix exponential e^{At}.

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answer:

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work:

Continued on back of page _____

[7] Let $A = \begin{bmatrix} -1 & 1 & 0 \\ -4 & 3 & 0 \\ 6 & -3 & 2 \end{bmatrix}$. Find the matrix exponential e^{At}.

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answer:

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work:

Continued on back of page _____

Final Exam

Linear Algebra, Dave Bayer, December 21, 2006

Name: _____

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (5 pts)	[**4**] (5 pts)	[**5**] (5 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Let $A = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$. Write A as CDC^{-1} for a diagonal matrix D. Find the matrix e^{At}.

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answer:

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work:

Continued on (back of) page _____

[2] Let $A = \begin{bmatrix} -3 & -2 \\ 6 & 4 \end{bmatrix}$. Write A as CDC^{-1} for a diagonal matrix D. Find the matrix e^{At}.

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answer:

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work:

[3] Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 1 & -1 \\ 2 & 2 & 1 \end{bmatrix}$. Write A as CDC^{-1} for a diagonal matrix D. Find the matrix e^{At}.

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answer:

---

work:

Continued on (back of) page _____

[4] Let $A = \begin{bmatrix} 8 & -1 \\ 1 & 6 \end{bmatrix}$. Find the matrix e^{At}.

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answer:

---

work:

Continued on (back of) page _____

[5] Let $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$. Find the matrix e^{At}.

---

answer:

---

work:

[**4**] Find the characteristic polynomial, and a system of eigenvalues and eigenvectors, for the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 6 & 0 \end{bmatrix}$$

[2] Find e^{At} for the matrix

$$A = \begin{bmatrix} -1 & -2 \\ 4 & 5 \end{bmatrix}$$

[3] Find e^{At} for the matrix

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

[**4**] Find A^n for the matrix

$$A = \begin{bmatrix} 2 & -4 \\ 1 & 6 \end{bmatrix}$$

[5] The quadratic form

$$2xy = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2} \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2} \left((x+y)^2 - (x-y)^2 \right)$$

can be expressed as shown as a linear combination of squares of linear forms. Do the same for the quadratic form

$$x^2 + 2xy + 2yz + z^2$$

[**4**] Find the matrix e^{At}, where $A = \begin{bmatrix} 2 & 2 & -2 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$.

[5] Find a matrix A so $A^2 = \begin{bmatrix} -2 & 6 \\ -3 & 7 \end{bmatrix}$.

[3] Find A^n for the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$$

[4] Find e^{At} for the matrix

$$A = \begin{bmatrix} 4 & -4 \\ 1 & 8 \end{bmatrix}$$

[5] Find e^{At} for the matrix

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

[**6**] Find e^{At} for the matrix

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

[**6**] Express the following quadratic form as a linear combination of squares of orthogonal linear forms:

$$3x^2 + 4xy + 6y^2$$

[7] Express the following quadratic form as a linear combination of squares of orthogonal linear forms:

$$2xy + 4xz + 4yz + 3z^2$$

[**8**] Find e^{At} for the matrix

$$A = \begin{bmatrix} 4 & 2 & 1 \\ -2 & -1 & -1 \\ -8 & -4 & -1 \end{bmatrix}$$

[5] Find the eigenvalues and corresponding eigenvectors of the matrix

$$\mathsf{A} \quad = \quad \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$$

[6] Find the matrix exponential e^{At}, for the matrix

$$A = \begin{bmatrix} 4 & 1 \\ 4 & 1 \end{bmatrix}$$

[7] Find the matrix exponential e^{At}, for the matrix

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

[8] Find a formula for A^n, for the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & -1 & 3 \end{bmatrix}$$