[4] Compute the determinant of the following 4×4 matrix:

$$\begin{bmatrix} \lambda & 1 & 0 & 0 \\ 1 & \lambda & 1 & 0 \\ 0 & 1 & \lambda & 1 \\ 0 & 0 & 1 & \lambda \end{bmatrix}$$

What can you say about the determinant of the $n \times n$ matrix with the same pattern?

 Continued on page: ______

[5] Use Cramer's rule to give a formula for w in the solution to the following system of equations:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Continued on page: ______

[4] Compute the determinant of the following 4×4 matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 2 & 0 & 2 \\ 3 & 0 & 3 & 3 \\ 0 & 4 & 4 & 4 \end{bmatrix}$$

What can you say about the determinant of the $n \times n$ matrix with the same pattern?

 Continued on page: ______

[5] Use Cramer's rule to give a formula for the solution to the following system of equations:

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2a \\ 2b \\ 2c \end{bmatrix}$$

 Continued on page: ______

Practive Exam 2

Linear Algebra, Dave Bayer, October 28, 1999

Name: ______________________________

ID: ______________ **School:** ______________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

To be graded, this practice exam must be turned in at the end of class on Thursday, November 4. Such exams will be returned in class on the following Tuesday, November 9. Participation is optional; scores will not be used to determine course grades. If you do participate, you may use your judgement in seeking any assistance of your choosing, or you may take this test under simulated exam conditions.

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] Let P be the set of all polynomials $f(x)$, and let Q be the subset of P consisting of all polynomials $f(x)$ so $f(0) = f(1) = 0$. Show that Q is a subspace of P.

 Continued on page: ______

[**4**] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which rotates one half turn around the axis given by the vector $(1,1,1)$. Find a matrix A representing L with respect to the standard basis

$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Choose a new basis $\{\mathbf{v}_1,\ \mathbf{v}_2, \mathbf{v}_3\}$ for $\mathbb{R}^3$ which makes L easier to describe, and find a matrix B representing L with respect to this new basis.

 Continued on page: ______

[**5**] Let $\{\mathbf{e}_1,\ \mathbf{e}_2\}$ and $\{\mathbf{v}_1,\ \mathbf{v}_2\}$ be ordered bases for $\mathbb{R}^2$, and let L be the linear transformation represented by the matrix A with respect to $\{\mathbf{e}_1,\ \mathbf{e}_2\}$, where

$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \quad A = \begin{bmatrix} 6 & -2 \\ -2 & 9 \end{bmatrix}.$$

Find the transition matrix S corresponding to the change of basis from $\{\mathbf{e}_1,\ \mathbf{e}_2\}$ to $\{\mathbf{v}_1,\ \mathbf{v}_2\}$. Find a matrix B representing L with respect to $\{\mathbf{v}_1,\ \mathbf{v}_2\}$.

Continued on page: ______

Exam 2

Linear Algebra, Dave Bayer, November 11, 1999

Name: ____

ID: ____________________ **School:** ____________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] Let P be the set of all degree ≤ 4 polynomials in one variable x with real coefficients. Let Q be the subset of P consisting of all odd polynomials, i.e. all polynomials $f(x)$ so $f(-x) = -f(x)$. Show that Q is a subspace of P. Choose a basis for Q. Extend this basis for Q to a basis for P.

 Continued on page: ______

[3] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which reflects through the plane P defined by $x + y + z = 0$. In other words, if $\mathbf{u}$ is a vector lying in the plane P, and $\mathbf{v}$ is a vector perpendicular to the plane P, then $L(\mathbf{u} + \mathbf{v}) = \mathbf{u} - \mathbf{v}$. Choose a basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for $\mathbb{R}^3$, and find a matrix A representing L with respect to this basis.

 Continued on page: ______

[4] Let $\{\mathbf{e}_1, \mathbf{e}_2\}$ and $\{\mathbf{v}_1, \mathbf{v}_2\}$ be ordered bases for $\mathbb{R}^2$, and let L be the linear transformation represented by the matrix A with respect to $\{\mathbf{e}_1, \mathbf{e}_2\}$, where

$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad A = \begin{bmatrix} -1 & 2 \\ -4 & 5 \end{bmatrix}.$$

Find the transition matrix S corresponding to the change of basis from $\{\mathbf{e}_1, \mathbf{e}_2\}$ to $\{\mathbf{v}_1, \mathbf{v}_2\}$. Find a matrix B representing L with respect to $\{\mathbf{v}_1, \mathbf{v}_2\}$.

 Continued on page: ______

[**5**] Let $\{\mathbf{u}_1,\ \mathbf{u}_2\}$, $\{\mathbf{v}_1,\ \mathbf{v}_2\}$, and $\{\mathbf{w}_1,\ \mathbf{w}_2\}$ be ordered bases for $\mathbb{R}^2$. If

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

is the transition matrix corresponding to the change of basis from $\{\mathbf{u}_1,\ \mathbf{u}_2\}$ to $\{\mathbf{v}_1,\ \mathbf{v}_2\}$, and

$$B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

is the transition matrix corresponding to the change of basis from $\{\mathbf{u}_1,\ \mathbf{u}_2\}$ to $\{\mathbf{w}_1,\ \mathbf{w}_2\}$, express $\mathbf{v}_1$ and $\mathbf{v}_2$ in terms of $\mathbf{w}_1$ and $\mathbf{w}_2$.

Continued on page: ______

Final Exam

Linear Algebra, Dave Bayer, December 16, 1999

Name: ______________________________

ID: ______________________ **School:** ______________________

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (5 pts)	[**4**] (5 pts)	
[**5**] (5 pts)	[**6**] (5 pts)	[**7**] (5 pts)	[**8**] (5 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which projects onto the line $(1,1,1)$. In other words, if $\mathbf{u}$ is a vector in $\mathbb{R}^3$, then $L(\mathbf{u})$ is the projection of $\mathbf{u}$ onto the vector $(1,1,1)$. Choose a basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for $\mathbb{R}^3$, and find a matrix A representing L with respect to this basis.

 Continued on page: ______

[2] Compute the determinant of the following 4×4 matrix:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

What can you say about the determinant of the $n \times n$ matrix with the same pattern?

Continued on page: ______

[3] By least squares, find the equation of the form $y = ax + b$ which best fits the data $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (1, 1)$, $(x_3, y_3) = (3, 1)$.

Continued on page: ______

[4] Find (s,t) so $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$ is as close as possible to $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

Continued on page: ______

[5] Find an orthogonal basis for the subspace $w + 2x + 3y + 4z = 0$ of $\mathbb{R}^4$.

Continued on page: ______

[3] Find the determinant of the matrix

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

What can you say about the determinant of the $n \times n$ matrix (n even) with the same pattern?

 Continued on page: ______

[4] Solve the following system of equations:

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

 Continued on page: ______

[5] Give a formula for the matrix which is inverse to:

$$\begin{bmatrix} \lambda & 1 & 1 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}$$

 Continued on page: ______

[3] Let V be the vector space of all polynomials $f(x)$ of degree ≤ 3. Let $W \subset V$ be the set of all polynomials f in V which satisfy $f'(1) = 1$. Is W is a subspace of V? Why or why not?

 Continued on (back of) page: ______

Final Exam

Linear Algebra, Dave Bayer, May 8, 2001

Name: ____

ID: ____________________ **School:** ____________________

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	[**6**] (6 pts)	[**7**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Let V be the vector space of all polynomials $f(x)$ of degree ≤ 3. Let $W \subset V$ be the subpace of all *odd* polynomials f in V, i.e. those polynomials $f(x)$ in V which satisfy $f(-x) = -f(x)$. Find a basis for W. Extend this basis to a basis for V.

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answer:

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work:

 Continued on back of page ______

[2] Find the determinants of the matrices

$$A_3 = \begin{bmatrix} \lambda & 0 & -1 \\ 0 & \lambda-1 & 0 \\ -1 & 0 & \lambda \end{bmatrix}, \quad A_4 = \begin{bmatrix} \lambda & 0 & 0 & -1 \\ 0 & \lambda & -1 & 0 \\ 0 & -1 & \lambda & 0 \\ -1 & 0 & 0 & \lambda \end{bmatrix}.$$

What is the determinant of the $n \times n$ matrix A_n with the same pattern?

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answer:

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work:

Continued on back of page ______

[**3**] Find an orthogonal basis for the subspace $w - x + y - z = 0$ of $\mathbb{R}^4$.

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answer:

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work:

Continued on back of page ______

[**4**] By least squares, find the equation of the form $y = ax + b$ which best fits the data $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (1, 2)$, $(x_3, y_3) = (2, 1)$.

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answer:

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work:

Continued on back of page ______

[5] Find (s,t) so $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$ is as close as possible to $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

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answer:

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work:

 Continued on back of page ______

[3] Compute the determinant of the following 4×4 matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{bmatrix}$$

What can you say about the determinant of the $n \times n$ matrix with the same pattern?

 Continued on page: ______

[4] Using Cramer's rule, find x satisfying the following system of equations:

$$\begin{bmatrix} 0 & s & t \\ s & t & s \\ t & s & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

 Continued on page: ______

[5] Give a formula for the matrix which is inverse to:

$$\begin{bmatrix} 1 & r & s \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix}$$

Continued on page: ______

[**3**] Let V be the vector space of all polynomials $f(x)$ of degree ≤ 3. Find a basis for the subspace W defined by $f(0) = f(1) = f(2)$. Extend this basis to a basis for V.

Continued on (back of) page: ______

Final Exam

Linear Algebra, Dave Bayer, May 13, 2003

Name: ________________________________

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	[**6**] (6 pts)	[**7**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Find an orthogonal basis for the subspace V of $\mathbb{R}^6$ consisting of all vectors (a, b, c, d, e, f) such that $a = b$, $c = d$, and $e = f$.

answer:

work:

 Continued on back of page ______

[2] Find an orthogonal basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors $(2, 1, 0, 0)$, $(0, 1, 1, 0)$, $(0, 0, 1, 2)$.

answer:

work:

Continued on back of page ______

[3] By least squares, find the equation of the form $y = ax + b$ which best fits the data $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (1, 2)$, $(x_3, y_3) = (2, 1)$, $(x_4, y_4) = (3, 0)$.

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answer:

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work:

 Continued on back of page ______

[4] Find (s,t) so $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$ is as close as possible to $\begin{bmatrix} 1 \\ 1 \\ 3 \\ 3 \end{bmatrix}$.

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answer:

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work:

Continued on back of page ______

[3] What is the determinant of the following 4×4 matrix?

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

 Continued on page: ______

[4] Using Cramer's rule, find w satisfying the following system of equations:

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Continued on page: ______

[5] Give a formula for the matrix which is inverse to:

$$\begin{bmatrix} a & 1 & 0 & 0 \\ 0 & b & 1 & 0 \\ 0 & 0 & c & 1 \\ 0 & 0 & 0 & d \end{bmatrix}$$

 Continued on page: ______

[**6**] What is the determinant of the following 8×8 matrix?

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 \end{bmatrix}$$

Continued on page: ______

[3] Let V be the vector space of all polynomials $f(x)$ of degree ≤ 3. Find a basis for the subspace W defined by

$$f(-1) = f(0), \quad f(-1) = f(1), \quad f(0) = f(1).$$

Extend this basis to a basis for V.

 Continued on (back of) page: ______

Final Exam

Linear Algebra, Dave Bayer, May 11, 2004

Name: ____

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	[**6**] (6 pts)	[**7**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Find an orthogonal basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors $(1, 0, 0, 1)$, $(0, 1, 0, 1)$, $(0, 0, 1, 1)$.

answer:

work:

Continued on back of page ______

[2] By least squares, find the equation of the form $y = ax + b$ which best fits the data $(x_1, y_1) = (-1, 0)$, $(x_2, y_2) = (0, 0)$, $(x_3, y_3) = (1, 1)$, $(x_4, y_4) = (2, 0)$.

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answer:

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work:

 Continued on back of page ______

[**3**] Find (s,t) so $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$ is as close as possible to $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$.

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answer:

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work:

Continued on back of page ______

[**3**] What is the determinant of the following 4×4 matrix?

$$\begin{bmatrix} 1 & 2 & a & c \\ 2 & 1 & b & d \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 4 & 3 \end{bmatrix}$$

 Continued on (back of) page: ______

[4] Using Cramer's rule, find w satisfying the following system of equations:

$$\begin{bmatrix} 1 & 3 & -1 & -1 \\ 1 & -1 & 3 & -1 \\ 1 & -1 & -1 & 3 \\ 1 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$

 Continued on (back of) page: ______

[5] Give a formula for the matrix which is inverse to:

$$\begin{bmatrix} 1 & a & 0 & -1 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

 Continued on (back of) page: ______

[6] What is the determinant of the following 6×6 matrix? What is the determinant of the corresponding $n \times n$ matrix?

$$\begin{bmatrix} 1 & 0 & x & 0 & 0 & 0 \\ 0 & 1 & 0 & x & 0 & 0 \\ 0 & 0 & 1 & 0 & x & 0 \\ 0 & 0 & 0 & 1 & 0 & x \\ x & 0 & 0 & 0 & 1 & 0 \\ 0 & x & 0 & 0 & 0 & 1 \end{bmatrix}$$

 Continued on (back of) page: ______

[3] Let V be the vector space of all polynomials $f(x)$ of degree ≤ 3. Find a basis for the subspace W defined by

$$f(1) = f'(1) = 0.$$

Extend this basis to a basis for V.

 Continued on (back of) page: ______

Final Exam

Linear Algebra, Dave Bayer, May 10, 2005

Name:

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	[**6**] (6 pts)	[**7**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Find an orthogonal basis for the subspace V of $\mathbb{R}^5$ spanned by the vectors

$$(1, 0, -1, 0, 1), \quad (1, 0, 0, -1, 1), \quad (0, 1, -1, 0, 1), \quad (0, 1, 0, -1, 1).$$

answer:

work:

 Continued on back of page

[2] By least squares, find the equation of the form $y = ax + b$ which best fits the data

$$(x_1, y_1) = (-1, 0), \quad (x_2, y_2) = (0, 0), \quad (x_3, y_3) = (0, 2), \quad (x_4, y_4) = (1, 1).$$

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answer:

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work:

Continued on back of page ______

[3] Find (s, t) so $\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$ is as close as possible to $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$.

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answer:

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work:

Continued on back of page ______

[4] What are the determinants of the following matrices? What is the determinant of the corresponding $n \times n$ matrix?

$$\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}, \quad \begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{bmatrix}, \quad \begin{bmatrix} 3 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}, \quad \begin{bmatrix} 3 & 1 & 0 & 0 & 0 \\ 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & 3 & 1 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

Continued on (back of) page: ______

[5] What is the determinant of the following matrix? What is the determinant of the corresponding $n \times n$ matrix?

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 1 \\ 3 & 3 & 3 & 2 & 1 \\ 4 & 4 & 3 & 2 & 1 \\ 5 & 4 & 3 & 2 & 1 \end{bmatrix}$$

 Continued on (back of) page: ______

Exam 2

Linear Algebra, Dave Bayer, October 19, 2006

Name: ______________________

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (5 pts)	[**4**] (5 pts)	[**5**] (5 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] What is the formula for the inverse to the following matrix?

$$\begin{bmatrix} A & B & D \\ 0 & C & E \\ 0 & 0 & F \end{bmatrix}$$

 Continued on (back of) page: ______

[2] Using Cramer's rule, solve for y in the following system of equations:

$$\begin{bmatrix} A & B & D \\ 0 & C & E \\ 0 & 0 & F \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Continued on (back of) page: ______

[4] Find a basis for the nullspace (homogeneous solutions) of the matrix shown. Extend this basis to a basis for all of $\mathbb{R}^5$.

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

 Continued on (back of) page: ______

Exam 3

Linear Algebra, Dave Bayer, November 16, 2006

Name: ______________________________

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (5 pts)	[**4**] (5 pts)	[**5**] (5 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] By least squares, find the equation of the form $y = ax + b$ which best fits the data

$$(x_1, y_1) = (0, 0), \quad (x_2, y_2) = (1, 0), \quad (x_3, y_3) = (2, 1), \quad (x_4, y_4) = (3, 0).$$

 Continued on (back of) page: ______

[**2**] Find (s,t) so $\begin{bmatrix} 3 & 1 \\ 2 & 2 \\ 1 & 3 \\ 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$ is as close as possible to $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

 Continued on (back of) page: ______

[3] Find an orthogonal basis for the subspace V of $\mathbb{R}^5$ spanned by the vectors

$$(1,0,-1,0,1), \quad (0,1,-1,1,0), \quad (1,-1,0,0,0), \quad (0,0,0,1,-1).$$

Continued on (back of) page: ______

[5] Define the inner product of two polynomials f and g by the rule

$$f \cdot g \quad = \quad \int_0^1 f(x)\, g(x)\, dx$$

Using this definition of the inner product, find an orthogonal basis for the vector space of all polynomials of degree ≤ 2.

 Continued on (back of) page: ______

Exam 2

Linear Algebra, Dave Bayer, March 20, 2007

Name: ____________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

Do not use calculators or decimal notation.

[1] Use Cramer's rule to solve for z in the system of equations

$$\begin{bmatrix} a & 1 & 0 \\ 1 & a & 1 \\ 0 & 1 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

[**5**] For each of the following matrices, find the determinant. What is the general pattern?

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Exam 3

Linear Algebra, Dave Bayer, April 17, 2007

Name: ______________________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

Do not use calculators or decimal notation. Please simplify each answer as far as possible.

[1] By least squares, find the equation of the form $z = ax + by + c$ which best fits the data

$$(x_1, y_1, z_1) = (0,0,0), \quad (x_2, y_2, z_2) = (1,0,0), \quad (x_3, y_3, z_3) = (0,1,0), \quad (x_4, y_4, z_4) = (1,1,1)$$

Exam 4

Linear Algebra, Dave Bayer, May 10, 2007

Name: ______________________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

Do not use calculators or decimal notation. Please simplify each answer as far as possible.

[1] Find an orthogonal basis for the subspace V of $\mathbb{R}^5$ spanned by the vectors

$$(\,1,\ 0, -1,\ 0,\ 1\,) \quad (\,0,\ 1, -1,\ 0,\ 0\,) \quad (\,0,\ 0,\ 1, -1,\ 0\,)$$

[**2**] Let V be the vector space of all polynomials $f(x)$ of degree $\leqslant 3$. Find a basis for the subspace W defined by

$$f(x) = f(-x)$$

Extend this basis to a basis for V.

[3] Define the inner product of two polynomials f and g by the rule

$$\langle f, g \rangle \quad = \quad \int_{-1}^{1} f(x)\, g(x)\, dx$$

Using this definition of the inner product, find an orthogonal basis for the vector space of all polynomials of degree $\leqslant 2$.

Exam 2

Linear Algebra, Dave Bayer, November 8, 2007

Name: ______________________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	[6] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

Do not use calculators or decimal notation.

[**1**] Use Cramer's rule to solve for x in the system of equations

$$\begin{bmatrix} a & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Exam 3 (Final)

Linear Algebra, Dave Bayer, December 20, 2007

Name: ____________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	[6] (5 pts)	[7] (5 pts)	[8] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

Do not use calculators or decimal notation.

[**1**] Find an orthogonal basis for the subspace V of $\mathbb{R}^4$ spanned by the rows of the matrix

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 3 & 3 \end{bmatrix}$$

[2] By least squares, find the equation of the form $z = ax + by + c$ which best fits the data

$$(x_1, y_1, z_1) = (0, 0, 1), \quad (x_2, y_2, z_2) = (1, 0, 1), \quad (x_3, y_3, z_3) = (0, 1, 0), \quad (x_4, y_4, z_4) = (1, 1, 2)$$

[3] Let V be the subspace of $\mathbb{R}^4$ spanned by the rows of the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 \end{bmatrix}$$

Find the matrix A which projects $\mathbb{R}^4$ orthogonally onto the subspace V.

[4] Let V be the vector space of all polynomials $f(x)$ of degree $\leqslant 3$. Find a basis for the subspace W defined by

$$f(0) = f(1) = f(2)$$

Extend this basis to a basis for V.

[5] Define the inner product of two polynomials f and g by the rule

$$\langle f, g \rangle \quad = \quad \int_0^1 f(x)\, g(1-x)\, dx$$

Using this definition of the inner product, find an orthogonal basis for the vector space of all polynomials of degree $\leqslant 2$.

[2] Find the determinant of each of the following matrices.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 3 & 4 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 9 \end{bmatrix} \qquad \begin{bmatrix} a & b & c & d \\ a & b+1 & c & d \\ a & b & c+1 & d \\ a & b & c & d+1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 4 \\ 1 & 2 & 6 & 4 \\ 1 & 2 & 3 & 8 \end{bmatrix}$$

[3] Find the inverse of the following matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ a & 1 & 0 & c \\ b & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[5] Find the ratio x/y for the solution to the matrix equation

$$\begin{bmatrix} a & d & 1 \\ b & e & 1 \\ c & f & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

[6] Find the determinant of the following 5×5 matrix. What is the determinant for the $n \times n$ case?

$$\begin{bmatrix} x & x^2 & 0 & 0 & 0 \\ 1 & x & x^2 & 0 & 0 \\ 0 & 1 & x & x^2 & 0 \\ 0 & 0 & 1 & x & x^2 \\ 0 & 0 & 0 & 1 & x \end{bmatrix}$$

Final Exam

Linear Algebra, Dave Bayer, May 10, 2011

Name: ______________________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	[6] (5 pts)	[7] (5 pts)	[8] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[1] By least squares, find the equation of the form $y = ax + b$ which best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 3 & 2 \end{bmatrix}$$

[2] Extend the vector (1,1,1,2) to an orthogonal basis for $\mathbb{R}^4$.

[3] Find the orthogonal projection of the vector (1,0,0,0) onto the subspace of $\mathbb{R}^4$ spanned by the vectors (1,1,1,0) and (0,1,1,1).

[4] Find the matrix A which projects $\mathbb{R}^4$ orthogonally onto the subspace spanned by the vectors (1,1,1,1) and (1,1,2,2).