Practive Exam 1

Linear Algebra, Dave Bayer, September 30, 1999

Name: ______________________________

ID: ____________________ **School:** ____________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

To be graded, this practice exam must be turned in at the end of class on Thursday, September 30. Such exams will be returned in class on the following Tuesday, October 5. Participation is optional; scores will not be used to determine course grades. If you do participate, you may use your judgement in seeking any assistance of your choosing, or you may take this test under simulated exam conditions. If you don't participate, you are electing to join the control group.

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

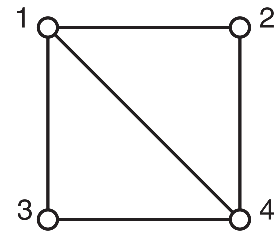
Please do not use calculators or decimal notation.

[**1**] Solve the following system of equations:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ -6 \end{bmatrix}$$

 Continued on page: ______

[2] Compute a matrix giving the number of walks of length 4 between pairs of vertices of the following graph:



 Continued on page: ______

[3] Express the following matrix as a product of elementary matrices:

$$\begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 \end{bmatrix}$$

Continued on page: ______

Exam 1

Linear Algebra, Dave Bayer, October 7, 1999

Name: ______________________________

ID: ____________________ **School:** ____________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

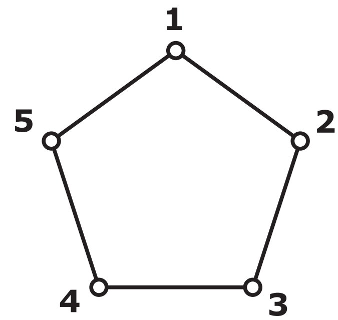
Please do not use calculators or decimal notation.

[**1**] Solve the following system of equations:

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

 Continued on page: ______

[2] Compute matrices giving the number of walks of lengths 1, 2, and 3 between pairs of vertices of the following graph:



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[**3**] Express the following matrix as a product of elementary matrices:

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

 Continued on page: ______

[2] Let A be the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 1 & -1 & 0 \end{bmatrix}.$$

Compute the row space and column space of A.

Continued on page: ______

[3] The four vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

span a subspace V of $\mathbb{R}^3$, but are not a basis for V. Choose a subset of $\{\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3,\ \mathbf{v}_4\}$ which forms a basis for V. Extend this basis for V to a basis for $\mathbb{R}^3$.

Continued on page: ______

[**2**] Let A be the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}.$$

Compute the row space and column space of A.

 Continued on page: ______

Exam 1

Linear Algebra, Dave Bayer, February 15, 2001

Name: ______________________________

ID: ____________________ **School:** ____________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] Solve the following system of equations:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 11 \end{bmatrix}$$

 Continued on page: ______

[2] Express the following matrix as a product of elementary matrices:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Continued on page: ______

Exam 2

Linear Algebra, Dave Bayer, March 29, 2001

Name: ____________________

ID: ____________________ **School:** ____________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Let A be the matrix

$$A \;=\; \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}.$$

Compute the row space and column space of A.

 Continued on (back of) page: ______

[2] Let

$$\mathbf{v}_1 = (1,1,0,-1), \quad \mathbf{v}_2 = (1,0,1,-1), \quad \mathbf{v}_3 = (0,1,1,-1), \quad \mathbf{v}_4 = (1,-1,0,0).$$

Find a basis for the subspace $V \subset \mathbb{R}^4$ spanned by $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$.

 Continued on (back of) page: ______

[4] Let $\mathbf{v}_1 = (1, 2)$ and $\mathbf{v}_2 = (1, 3)$. Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that

$$L(\mathbf{v}_1) = \mathbf{v}_1, \quad L(\mathbf{v}_2) = \mathbf{v}_1 + \mathbf{v}_2.$$

Find a matrix that represents L with respect to the usual basis $\mathbf{e}_1 = (1, 0)$, $\mathbf{e}_2 = (0, 1)$.

 Continued on (back of) page: ______

[5] Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation such that $L(\mathbf{v}) = \mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace $V \subset \mathbb{R}^3$ defined by $x + y = z$, and $L(\mathbf{v}) = \mathbf{0}$ for all $\mathbf{v}$ belonging to the subspace $W \subset \mathbb{R}^3$ defined by $x = y = z$. Find a matrix that represents L with respect to the usual basis

$$\mathbf{e}_1 = (1, 0, 0), \quad \mathbf{e}_2 = (0, 1, 0), \quad \mathbf{e}_3 = (0, 0, 1).$$

 Continued on (back of) page: ______

Exam 1

Linear Algebra, Dave Bayer, February 20, 2003

Name: ______________________________

ID: ______________________ **School:** ______________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] Solve the following system of equations:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Continued on page: ______

[2] Express the following matrix as a product of elementary matrices:

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Continued on page: ______

Exam 2

Linear Algebra, Dave Bayer, April 3, 2003

Name: ________________________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Let A be the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 & -3 & 5 & -7 \\ -1 & 2 & -3 & 5 & -7 & 12 \\ 2 & -3 & 5 & -7 & 12 & -19 \end{bmatrix}.$$

Compute the row space and column space of A.

 Continued on (back of) page: ______

[2] Let

$$\mathbf{v}_1 = (1, 2, -3, -4), \quad \mathbf{v}_2 = (1, -2, 3, -4), \quad \mathbf{v}_3 = (0, 2, -3, 0), \quad \mathbf{v}_4 = (1, -2, -3, 4).$$

Find a basis for the subspace $V \subset \mathbb{R}^4$ spanned by $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$. Extend this basis to a basis for $\mathbb{R}^4$.

 Continued on (back of) page: ______

[4] Let $\mathbf{v}_1 = (1,1)$ and $\mathbf{v}_2 = (1,2)$. Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that

$$L(\mathbf{v}_1) = \mathbf{v}_1 + \mathbf{v}_2, \quad L(\mathbf{v}_2) = \mathbf{v}_1 - \mathbf{v}_2.$$

Find a matrix that represents L with respect to the usual basis $\mathbf{e}_1 = (1,0)$, $\mathbf{e}_2 = (0,1)$.

Continued on (back of) page: ______

[**5**] Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation such that $L(\mathbf{v}) = \mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace $V \subset \mathbb{R}^3$ defined by $x + y = 2z$, and $L(\mathbf{v}) = 2\mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace $W \subset \mathbb{R}^3$ defined by $x = y = 2z$. Find a matrix that represents L with respect to the usual basis

$$\mathbf{e}_1 = (1, 0, 0), \quad \mathbf{e}_2 = (0, 1, 0), \quad \mathbf{e}_3 = (0, 0, 1).$$

Continued on (back of) page: ______

Exam 1

Linear Algebra, Dave Bayer, February 17, 2004

Name: ____________________

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (5 pts)	[**4**] (5 pts)	[**5**] (5 pts)	[**6**] (5 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] What is the set of all solutions to the following system of equations?

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Continued on page: ______

[2] Express the following matrix as a product of elementary matrices:

$$\begin{bmatrix} 0 & 0 & 2 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

 Continued on page: ______

Second Midterm

Linear Algebra, Dave Bayer, March 30, 2004

Name: ______________________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Let A be the matrix

$$A \;=\; \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 2 & 0 \\ -1 & 0 & -1 & -1 & -1 & 0 \\ 0 & -1 & -1 & 1 & -2 & 1 \end{bmatrix}.$$

Compute the row space and column space of A.

 Continued on (back of) page: ______

[2] Let

$$\mathbf{v}_1 = (1,1,0,0),\ \mathbf{v}_2 = (1,0,1,0),\ \mathbf{v}_3 = (1,0,0,-1),\ \mathbf{v}_4 = (0,1,-1,0),\ \mathbf{v}_5 = (0,1,0,1).$$

Find a basis for the subspace $V \subset \mathbb{R}^4$ spanned by $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$, and $\mathbf{v}_5$. Extend this basis to a basis for $\mathbb{R}^4$.

 Continued on (back of) page: ______

[4] Let $\mathbf{v}_1 = (1, -1)$ and $\mathbf{v}_2 = (1, 1)$. Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that

$$L(\mathbf{v}_1) = 3\mathbf{v}_1 - \mathbf{v}_2, \quad L(\mathbf{v}_2) = 3\mathbf{v}_2 - \mathbf{v}_1.$$

Find a matrix that represents L with respect to the usual basis $\mathbf{e}_1 = (1, 0)$, $\mathbf{e}_2 = (0, 1)$.

 Continued on (back of) page: ______

[5] Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation such that $L(\mathbf{v}) = -\mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace $V \subset \mathbb{R}^3$ defined by $x + y + z = 0$, and $L(\mathbf{v}) = \mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace $W \subset \mathbb{R}^3$ defined by $x = y = 0$. Find a matrix that represents L with respect to the usual basis

$$\mathbf{e}_1 = (1, 0, 0), \quad \mathbf{e}_2 = (0, 1, 0), \quad \mathbf{e}_3 = (0, 0, 1).$$

Continued on (back of) page: ______

Exam 1

Linear Algebra, Dave Bayer, February 15, 2005

Name: ______________________________

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (5 pts)	[**4**] (5 pts)	[**5**] (5 pts)	[**6**] (5 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] What is the set of all solutions to the following system of equations?

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 3 \\ 3 \end{bmatrix}$$

 Continued on (back of) page: ______

[2] Express the following matrix as a product of elementary matrices:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ -2 & 2 & -3 \end{bmatrix}$$

 Continued on (back of) page: ______

Second Midterm

Linear Algebra, Dave Bayer, March 29, 2005

Name: ______________________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Let A be the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 2 & 3 & 5 \\ 1 & 1 & 2 & 3 & 5 & 8 \\ 1 & 2 & 3 & 5 & 8 & 13 \\ 1 & 3 & 5 & 8 & 13 & 21 \end{bmatrix}.$$

Compute the row space and column space of A.

 Continued on (back of) page: ______

[2] Let

$$\mathbf{v}_1 = (1,1,0,1),\ \mathbf{v}_2 = (1,0,-1,0),\ \mathbf{v}_3 = (1,-3,0,-1),\ \mathbf{v}_4 = (0,1,-1,0),\ \mathbf{v}_5 = (0,1,1,1).$$

Find a basis for the subspace $V \subset \mathbb{R}^4$ spanned by $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$, and $\mathbf{v}_5$. Extend this basis to a basis for $\mathbb{R}^4$.

 Continued on (back of) page: ______

[4] Let $\mathbf{v}_1 = (1, 1)$ and $\mathbf{v}_2 = (1, 2)$. Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that

$$L(\mathbf{v}_1) = \mathbf{v}_1 + 2\mathbf{v}_2, \quad L(\mathbf{v}_2) = \mathbf{v}_1 + \mathbf{v}_2.$$

Find a matrix that represents L with respect to the usual basis $\mathbf{e}_1 = (1, 0)$, $\mathbf{e}_2 = (0, 1)$.

Continued on (back of) page: ______

[5] Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation such that $L(\mathbf{v}) = 2\mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace $V \subset \mathbb{R}^3$ defined by $x + y + z = 0$, and $L(1, 1, 1) = (1, 0, -1)$. Find a matrix that represents L with respect to the usual basis $\mathbf{e}_1 = (1, 0, 0)$, $\mathbf{e}_2 = (0, 1, 0)$, $\mathbf{e}_3 = (0, 0, 1)$.

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Exam 1

Linear Algebra, Dave Bayer, September 26, 2006

Name: ____________________

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (5 pts)	[**4**] (5 pts)	[**5**] (5 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] What is the set of all solutions to the following system of equations?

$$\begin{bmatrix} 2 & 1 & 4 \\ 1 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

 Continued on (back of) page: ______

[2] What is the set of all solutions to the following system of equations?

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \\ -1 \end{bmatrix}$$

 Continued on (back of) page: ______

[3] Express the following matrix as a product of elementary matrices:

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 8 \end{bmatrix}$$

Continued on (back of) page: ______

[3] Find a basis for the rowspace, and find a basis for the column space, of the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 1 & 2 \\ 1 & 2 & 0 & 1 & 2 & 0 \\ 2 & 0 & 1 & 2 & 0 & 1 \end{bmatrix}$$

Continued on (back of) page: ______

[5] Let A be the 3×3 matrix determined by

$$A\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \qquad A\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}, \qquad A\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

Find A.

 Continued on (back of) page: ______

[**4**] Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation such that $L(\mathbf{v}) = \mathbf{v}$ for all $\mathbf{v}$ belonging to the subspace defined by $x - y + z = 0$, and $L(\mathbf{v}) = 0$ for all $\mathbf{v}$ belonging to the subspace defined by $x = y = 0$. Find a matrix that represents L with respect to the usual basis $\mathbf{e}_1 = (1, 0, 0)$, $\mathbf{e}_2 = (0, 1, 0)$, $\mathbf{e}_3 = (0, 0, 1)$.

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Exam 1

Linear Algebra, Dave Bayer, February 13, 2007

Name: ____________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

Do not use calculators or decimal notation.

[1] What is the set of all solutions to the following system of equations?

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

[2] What is the set of all solutions to each of the following systems of equations?

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 2 & 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \qquad \begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 2 & 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

[3] Use Gaussian elimination to find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

[4] Express A as a product of elementary matrices, where

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 3 \end{bmatrix}$$

[5] Express A^{-1} as a product of elementary matrices, where

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

[**3**] Find a 3×3 matrix A such that

$$A \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \qquad A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \qquad A \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}$$

Exam 1

Linear Algebra, Dave Bayer, October 2, 2007

Name: ______________________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

Do not use calculators or decimal notation.

[1] Use Gaussian elimination to find the inverse of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

[2] What is the set of all solutions to the following system of equations?

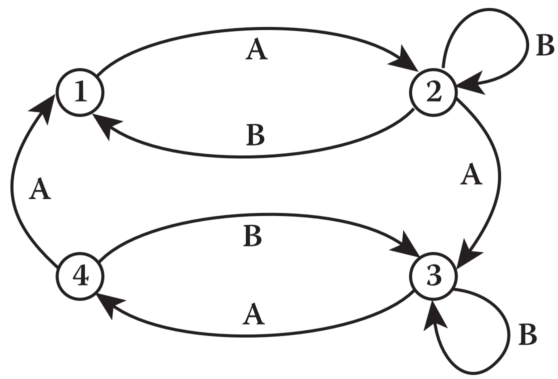
$$\begin{bmatrix} 0 & 1 & 1 & 0 & 2 & 0 & 4 \\ 0 & 0 & 0 & 1 & 3 & 0 & 5 \\ 0 & 1 & 1 & 0 & 2 & 1 & 10 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 16 \end{bmatrix}$$

[**3**] Express A as a product of elementary matrices, where

$$A = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

[4] Find a matrix representing the linear map from $\mathbb{R}^2$ to $\mathbb{R}^2$ which reflects first across the line $y = x$, then across the line $y = 2x$.

[5] Compute a matrix giving the number of walks of length 4 between pairs of vertices of the following directed graph:



How many of these paths are labeled ABAB ?

[2] Find a basis for the subspace V of $\mathbb{R}^5$ defined by the following system of equations. Extend this basis to a basis for all of $\mathbb{R}^5$.

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 5 \\ 0 & 1 & 0 & 3 & 6 \\ 0 & 1 & 0 & 3 & 6 \\ 0 & 0 & 1 & 4 & 7 \end{bmatrix} \begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Exam 1

Linear Algebra, Dave Bayer, February 15, 2011

Name: ______________________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	[6] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[1] What is the set of all solutions to the following system of equations?

$$\begin{bmatrix} 3 & 1 & 1 \\ 2 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ 5 \end{bmatrix}$$

[2] What is the set of all solutions to the following system of equations?

$$\begin{bmatrix} 0 & 1 & 1 & 5 & 0 & 9 \\ 0 & 0 & 1 & 3 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 6 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix}$$

[3] Use Gaussian elimination to find the inverse of the matrix

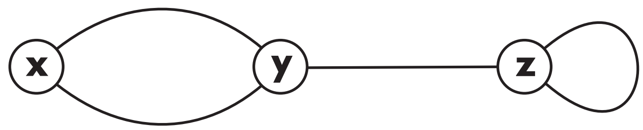
$$A = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 3 & 0 \end{bmatrix}$$

[4] Express A as a product of elementary matrices, where

$$A = \begin{bmatrix} 0 & 3 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

[5] Let $A : \mathbb{R}^2 \to \mathbb{R}^2$ be the matrix which flips the plane $\mathbb{R}^2$ across the line $3x = y$. Find A.

[6] Using matrix multiplication, count the number of paths of length four from y to itself.



Exam 2

Linear Algebra, Dave Bayer, March 29, 2011

Name: ______________________________

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	[6] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[1] Find a basis for the rowspace of the following matrix. Extend this basis to a basis for all of $\mathbb{R}^4$.

$$\begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

[4] Let

$$v_1 = (1,0,0), \qquad v_2 = (1,1,0), \qquad v_3 = (0,1,1)$$

Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear map such that

$$L(v_1) = v_2, \qquad L(v_2) = v_3, \qquad L(v_3) = v_1,$$

Find the matrix A (in standard coordinates) which represents the linear map L.