

# Final Exam

Linear Algebra, Dave Bayer, May 10, 2011

Name: \_\_\_\_\_

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	[6] (5 pts)	[7] (5 pts)	[8] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[1] By least squares, find the equation of the form  $y = ax + b$  which best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 3 & 2 \end{bmatrix}$$

**[2]** Extend the vector  $(1,1,1,2)$  to an orthogonal basis for  $\mathbb{R}^4$ .

**[3]** Find the orthogonal projection of the vector  $(1,0,0,0)$  onto the subspace of  $\mathbb{R}^4$  spanned by the vectors  $(1,1,1,0)$  and  $(0,1,1,1)$ .

[4] Find the matrix  $A$  which projects  $\mathbb{R}^4$  orthogonally onto the subspace spanned by the vectors  $(1,1,1,1)$  and  $(1,1,2,2)$ .

[5] Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$$

[6] Find the matrix exponential  $e^{At}$ , for the matrix

$$A = \begin{bmatrix} 4 & 1 \\ 4 & 1 \end{bmatrix}$$

[7] Find the matrix exponential  $e^{At}$ , for the matrix

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

[8] Find a formula for  $A^n$ , for the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & -1 & 3 \end{bmatrix}$$