Exam 2

Linear Algebra, Dave Bayer, March 29, 2011

Name: _____

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	[6] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[1] Find a basis for the rowspace of the following matrix. Extend this basis to a basis for all of $\mathbb{R}^4$.

$$\begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

[2] Find the determinant of each of the following matrices.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 3 & 4 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 9 \end{bmatrix} \qquad \begin{bmatrix} a & b & c & d \\ a & b+1 & c & d \\ a & b & c+1 & d \\ a & b & c & d+1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 4 \\ 1 & 2 & 6 & 4 \\ 1 & 2 & 3 & 8 \end{bmatrix}$$

[3] Find the inverse of the following matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \mathfrak{a} & 1 & 0 & \mathfrak{c} \\ \mathfrak{b} & 0 & 1 & \mathfrak{d} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[4] Let

$$v_1 = (1,0,0), \qquad v_2 = (1,1,0), \qquad v_3 = (0,1,1)$$

Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear map such that

$$L(v_1) = v_2, \qquad L(v_2) = v_3, \qquad L(v_3) = v_1,$$

Find the matrix A (in standard coordinates) which represents the linear map L.

[5] Find the ratio x/y for the solution to the matrix equation

$$\begin{bmatrix} a & d & 1 \\ b & e & 1 \\ c & f & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

[6] Find the determinant of the following 5×5 matrix. What is the determinant for the $n \times n$ case?

$$\begin{bmatrix} x & x^2 & 0 & 0 & 0 \\ 1 & x & x^2 & 0 & 0 \\ 0 & 1 & x & x^2 & 0 \\ 0 & 0 & 1 & x & x^2 \\ 0 & 0 & 0 & 1 & x \end{bmatrix}$$