

--	--	--

## Exam 3

Linear Algebra, Dave Bayer, April 17, 2007

Name: \_\_\_\_\_

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

Do not use calculators or decimal notation. Please simplify each answer as far as possible.

[1] By least squares, find the equation of the form  $z = ax + by + c$  which best fits the data

$$(x_1, y_1, z_1) = (0,0,0), \quad (x_2, y_2, z_2) = (1,0,0), \quad (x_3, y_3, z_3) = (0,1,0), \quad (x_4, y_4, z_4) = (1,1,1)$$

[2] Find  $e^{At}$  for the matrix

$$A = \begin{bmatrix} -1 & -2 \\ 4 & 5 \end{bmatrix}$$

[3] Find  $e^{At}$  for the matrix

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

[4] Find  $A^n$  for the matrix

$$A = \begin{bmatrix} 2 & -4 \\ 1 & 6 \end{bmatrix}$$

[5] The quadratic form

$$2xy = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2} \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2} ((x+y)^2 - (x-y)^2)$$

can be expressed as shown as a linear combination of squares of linear forms. Do the same for the quadratic form

$$x^2 + 2xy + 2yz + z^2$$