

## Exam 2

Linear Algebra, Dave Bayer, April 3, 2003

Name: \_\_\_\_\_

[1] (6 pts)	[2] (6 pts)	[3] (6 pts)	[4] (6 pts)	[5] (6 pts)	TOTAL

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[1] Let  $A$  be the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 & -3 & 5 & -7 \\ -1 & 2 & -3 & 5 & -7 & 12 \\ 2 & -3 & 5 & -7 & 12 & -19 \end{bmatrix}.$$

Compute the row space and column space of  $A$ .

[2] Let

$$\mathbf{v}_1 = (1, 2, -3, -4), \quad \mathbf{v}_2 = (1, -2, 3, -4), \quad \mathbf{v}_3 = (0, 2, -3, 0), \quad \mathbf{v}_4 = (1, -2, -3, 4).$$

Find a basis for the subspace  $V \subset \mathbb{R}^4$  spanned by  $\mathbf{v}_1$ ,  $\mathbf{v}_2$ ,  $\mathbf{v}_3$ , and  $\mathbf{v}_4$ . Extend this basis to a basis for  $\mathbb{R}^4$ .

[3] Let  $V$  be the vector space of all polynomials  $f(x)$  of degree  $\leq 3$ . Find a basis for the subspace  $W$  defined by  $f(0) = f(1) = f(2)$ . Extend this basis to a basis for  $V$ .

[4] Let  $\mathbf{v}_1 = (1, 1)$  and  $\mathbf{v}_2 = (1, 2)$ . Let  $L : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the linear transformation such that

$$L(\mathbf{v}_1) = \mathbf{v}_1 + \mathbf{v}_2, \quad L(\mathbf{v}_2) = \mathbf{v}_1 - \mathbf{v}_2.$$

Find a matrix that represents  $L$  with respect to the usual basis  $\mathbf{e}_1 = (1, 0)$ ,  $\mathbf{e}_2 = (0, 1)$ .

[5] Let  $L : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the linear transformation such that  $L(\mathbf{v}) = \mathbf{v}$  for all  $\mathbf{v}$  belonging to the subspace  $V \subset \mathbb{R}^3$  defined by  $x + y = 2z$ , and  $L(\mathbf{v}) = 2\mathbf{v}$  for all  $\mathbf{v}$  belonging to the subspace  $W \subset \mathbb{R}^3$  defined by  $x = y = 2z$ . Find a matrix that represents  $L$  with respect to the usual basis

$$\mathbf{e}_1 = (1, 0, 0), \quad \mathbf{e}_2 = (0, 1, 0), \quad \mathbf{e}_3 = (0, 0, 1).$$

**Problem:** \_\_\_\_\_