# Final Exam

Linear Algebra, Dave Bayer, May 13, 2003

**Name:** ________________________________________

| [**1**] (5 pts) | [**2**] (5 pts) | [**3**] (6 pts) | [**4**] (6 pts) | [**5**] (6 pts) | [**6**] (6 pts) | [**7**] (6 pts) | **TOTAL** |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

Please work only one problem per page, starting with the pages provided, and identify all continuations clearly.

[**1**] Find an orthogonal basis for the subspace $V$ of $\mathbb{R}^6$ consisting of all vectors $(a, b, c, d, e, f)$ such that $a = b$, $c = d$, and $e = f$.

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*answer:*

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*work:*



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**[2]** Find an orthogonal basis for the subspace $V$ of $\mathbb{R}^4$ spanned by the vectors $(2, 1, 0, 0)$, $(0, 1, 1, 0)$, $(0, 0, 1, 2)$.

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*answer:*

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*work:*



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**[3]** By least squares, find the equation of the form $y = ax + b$ which best fits the data $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (1, 2)$, $(x_3, y_3) = (2, 1)$, $(x_4, y_4) = (3, 0)$.

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*answer:*

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*work:*



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[**4**] Find $(s,t)$ so $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$ is as close as possible to $\begin{bmatrix} 1 \\ 1 \\ 3 \\ 3 \end{bmatrix}$.

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*answer:*

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*work:*



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**[5]** Let $A = \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 0 \\ 1 & 1 & -2 \end{bmatrix}$. Find the eigenvalues and eigenvectors of $A$.

*answer:*

*work:*



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**[6]** Let $A = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$. Find the matrix exponential $e^{At}$.

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*answer:*

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*work:*



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[**7**] Let $A = \begin{bmatrix} 0 & 1 & -1 \\ -2 & 3 & -1 \\ -2 & 2 & 0 \end{bmatrix}$. Find the matrix exponential $e^{At}$.

*answer:*

*work:*



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