Final Exam

Linear Algebra, Dave Bayer, December 22, 2022

Name: ______________________________ **Uni:** ______________

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	**Total**

If you need more that one page for a problem, clearly indicate on each page where to look next for your work.

[1] Find a system of equations having as solution set the following affine subspace of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -2 \\ 1 \end{bmatrix} t$$

$$\begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$$

[2] Find the 3×3 matrix A that projects $\mathbb{R}^3$ orthogonally onto the hyperplane $x + y + 3z = 0$.

$$A = \frac{1}{\square} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

[3] Find the 3×3 matrix A that projects $\mathbb{R}^3$ orthogonally onto the plane $x + y + z = 0$, with respect to the inner product

$$< (a, b, c),\ (r, s, t) > \quad = \quad \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} r \\ s \\ t \end{bmatrix}$$

$$A \;=\; \frac{1}{\square} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix}$$

$$A^n = \frac{\square}{\square}\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

[5] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$y = \frac{\square}{\square}\begin{bmatrix} \square \\ \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square \\ \square \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$$

$$e^{At} = \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

[7] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$e^{At} = \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

[8] Express the quadratic form

$$-2x^2 - 2xy - 3y^2 - 2xz - 3z^2$$

as a sum of squares of orthogonal linear forms.

$$\square\left(\qquad\right)^2 + \square\left(\qquad\right)^2 + \square\left(\qquad\right)^2$$