Final Exam

Linear Algebra, Dave Bayer, December 16, 2022

[1] Find a system of equations having as solution set the following affine subspace of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix} t$$

[2] Find the 3×3 matrix A that projects $\mathbb{R}^3$ orthogonally onto the plane $x + y + 2z = 0$.

[3] Find the 3×3 matrix A that projects $\mathbb{R}^3$ orthogonally onto the plane $x + y + z = 0$, with respect to the inner product

$$< (a, b, c),\ (r, s, t) > \quad = \quad \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} r \\ s \\ t \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix}$$

[5] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

[7] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

[8] Express the quadratic form

$$3x^2 - 2xy + 2y^2 - 2xz + 2z^2$$

as a linear combination of squares of orthogonal linear forms.