



# Final Exam

Linear Algebra, Dave Bayer, December 16, 2022

Name: \_\_\_\_\_ Uni: \_\_\_\_\_

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	Total

If you need more than one page for a problem, clearly indicate on each page where to look next for your work.

[1] Find a system of equations having as solution set the following affine subspace of  $\mathbb{R}^4$ .

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix} t$$

$$\begin{bmatrix} \boxed{\phantom{0}} & \boxed{\phantom{0}} & \boxed{\phantom{0}} & \boxed{\phantom{0}} \\ \boxed{\phantom{0}} & \boxed{\phantom{0}} & \boxed{\phantom{0}} & \boxed{\phantom{0}} \\ \boxed{\phantom{0}} & \boxed{\phantom{0}} & \boxed{\phantom{0}} & \boxed{\phantom{0}} \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \boxed{\phantom{0}} \\ \boxed{\phantom{0}} \\ \boxed{\phantom{0}} \end{bmatrix}$$



[2] Find the  $3 \times 3$  matrix  $A$  that projects  $\mathbb{R}^3$  orthogonally onto the plane  $x + y + 2z = 0$ .

$$A = \frac{1}{\boxed{\phantom{000}}} \begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$



[3] Find the  $3 \times 3$  matrix  $A$  that projects  $\mathbb{R}^3$  orthogonally onto the plane  $x + y + z = 0$ , with respect to the inner product

$$\langle (a, b, c), (r, s, t) \rangle = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} r \\ s \\ t \end{bmatrix}$$

$$A = \frac{1}{\boxed{\phantom{000}}} \begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$



[4] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix}$$

$$A^n = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix} + \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$



[5] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$y = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \begin{bmatrix} \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \end{bmatrix} + \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \begin{bmatrix} \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \end{bmatrix}$$



[6] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$e^{At} = \frac{\begin{bmatrix} \phantom{0} \\ \phantom{0} \end{bmatrix}}{\begin{bmatrix} \phantom{0} \\ \phantom{0} \end{bmatrix}} \begin{bmatrix} \phantom{0} & \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} \end{bmatrix} + \frac{\begin{bmatrix} \phantom{0} \\ \phantom{0} \end{bmatrix}}{\begin{bmatrix} \phantom{0} \\ \phantom{0} \end{bmatrix}} \begin{bmatrix} \phantom{0} & \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} \end{bmatrix} + \frac{\begin{bmatrix} \phantom{0} \\ \phantom{0} \end{bmatrix}}{\begin{bmatrix} \phantom{0} \\ \phantom{0} \end{bmatrix}} \begin{bmatrix} \phantom{0} & \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} \end{bmatrix}$$



[7] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$e^{At} = \frac{\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}}{\begin{array}{|c|} \hline \square \\ \hline \end{array}} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}}{\begin{array}{|c|} \hline \square \\ \hline \end{array}} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}}{\begin{array}{|c|} \hline \square \\ \hline \end{array}} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

$$3x^2 - 2xy + 2y^2 - 2xz + 2z^2$$

$$\boxed{\phantom{00}} \left( \boxed{\phantom{000000}} \right)^2 + \boxed{\phantom{00}} \left( \boxed{\phantom{000000}} \right)^2 + \boxed{\phantom{00}} \left( \boxed{\phantom{000000}} \right)^2$$