2×2 Exercise Set A (distinct roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find A^n where A is the matrix

$$A = \begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix}$$

$$\lambda = -2, 0 \qquad A^n = \frac{(-2)^n}{2} \begin{bmatrix} -1 & 1 \\ -3 & 3 \end{bmatrix} + \frac{0^n}{2} \begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$A = \begin{bmatrix} 1 & -1 \\ -3 & -1 \end{bmatrix}$$

$$\lambda = -2, 2 \qquad A^n = \frac{(-2)^n}{4} \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} + \frac{2^n}{4} \begin{bmatrix} 3 & -1 \\ -3 & 1 \end{bmatrix}$$

[3] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

$$\lambda = -1, 2 \qquad A^n = \frac{(-1)^n}{3} \begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix} + \frac{2^n}{3} \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

$$\lambda = -2, 3 \qquad A^n = \frac{(-2)^n}{5} \begin{bmatrix} 2 & -2 \\ -3 & 3 \end{bmatrix} + \frac{3^n}{5} \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$$

[5] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 2 \\ -2 & -3 \end{bmatrix}$$

$$\lambda = -2, 1 \qquad A^n = \frac{(-2)^n}{3} \begin{bmatrix} -1 & -2 \\ 2 & 4 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 4 & 2 \\ -2 & -1 \end{bmatrix}$$

[6] Find A^n where A is the matrix

$$A = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}$$

$$\lambda = -2, -1 \qquad A^n \;=\; (-2)^n \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} + (-1)^n \begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix}$$

[7] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$\lambda = -1, 2 \qquad A^n \;=\; \frac{(-1)^n}{3} \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} + \frac{2^n}{3} \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$$

[8] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$\lambda = -2, -1 \qquad A^n \;=\; (-2)^n \begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} + (-1)^n \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix}$$

2×2 Exercise Set B (distinct roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 3 & 1 \\ -3 & -1 \end{bmatrix}$$

$$\lambda = 0, 2 \qquad e^{At} = \frac{1}{2}\begin{bmatrix} -1 & -1 \\ 3 & 3 \end{bmatrix} + \frac{e^{2t}}{2}\begin{bmatrix} 3 & 1 \\ -3 & -1 \end{bmatrix}$$

[2] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -3 & 2 \\ -3 & 2 \end{bmatrix}$$

$$\lambda = -1, 0 \qquad e^{At} = e^{-t}\begin{bmatrix} 3 & -2 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} -2 & 2 \\ -3 & 3 \end{bmatrix}$$

[3] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -1 & -3 \\ -2 & 0 \end{bmatrix}$$

$$\lambda = -3, 2 \qquad e^{At} = \frac{e^{-3t}}{5}\begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix} + \frac{e^{2t}}{5}\begin{bmatrix} 2 & -3 \\ -2 & 3 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 3 & 3 \\ -1 & -1 \end{bmatrix}$$

$$\lambda = 0, 2 \qquad e^{At} = \frac{1}{2}\begin{bmatrix} -1 & -3 \\ 1 & 3 \end{bmatrix} + \frac{e^{2t}}{2}\begin{bmatrix} 3 & 3 \\ -1 & -1 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & -1 \\ -3 & -1 \end{bmatrix}$$

$$\lambda = -2, 2 \qquad e^{At} = \frac{e^{-2t}}{4}\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} + \frac{e^{2t}}{4}\begin{bmatrix} 3 & -1 \\ -3 & 1 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -2 & 2 \\ -3 & 3 \end{bmatrix}$$

$$\lambda = 0, 1 \qquad e^{At} = \begin{bmatrix} 3 & -2 \\ 3 & -2 \end{bmatrix} + e^{t} \begin{bmatrix} -2 & 2 \\ -3 & 3 \end{bmatrix}$$

[7] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & -2 \\ 3 & -3 \end{bmatrix}$$

$$\lambda = -1, 0 \qquad e^{At} = e^{-t} \begin{bmatrix} -2 & 2 \\ -3 & 3 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 3 & -2 \end{bmatrix}$$

[8] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & -1 \\ -3 & 2 \end{bmatrix}$$

$$\lambda = -1, 3 \qquad e^{At} = \frac{e^{-t}}{4} \begin{bmatrix} 3 & 1 \\ 3 & 1 \end{bmatrix} + \frac{e^{3t}}{4} \begin{bmatrix} 1 & -1 \\ -3 & 3 \end{bmatrix}$$

2×2 Exercise Set C (distinct roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Solve the differential equation $y' = Ay$ where

$$A \;=\; \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, \qquad y(0) \;=\; \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\lambda = 0, 3 \qquad e^{At} \;=\; \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix} + \frac{e^{3t}}{3}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \qquad y \;=\; \frac{1}{3}\begin{bmatrix} 2 \\ -2 \end{bmatrix} + \frac{e^{3t}}{3}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

[2] Solve the differential equation $y' = Ay$ where

$$A \;=\; \begin{bmatrix} 0 & -1 \\ 2 & -3 \end{bmatrix}, \qquad y(0) \;=\; \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\lambda = -2, -1 \qquad e^{At} \;=\; e^{-2t}\begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix} + e^{-t}\begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \qquad y \;=\; e^{-2t}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + e^{-t}\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

[3] Solve the differential equation $y' = Ay$ where

$$A \;=\; \begin{bmatrix} 1 & -1 \\ -3 & -1 \end{bmatrix}, \qquad y(0) \;=\; \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda = -2, 2 \qquad e^{At} \;=\; \frac{e^{-2t}}{4}\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} + \frac{e^{2t}}{4}\begin{bmatrix} 3 & -1 \\ -3 & 1 \end{bmatrix} \qquad y \;=\; \frac{e^{-2t}}{4}\begin{bmatrix} 2 \\ 6 \end{bmatrix} + \frac{e^{2t}}{4}\begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

[4] Solve the differential equation $y' = Ay$ where

$$A \;=\; \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}, \qquad y(0) \;=\; \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\lambda = -3, 0 \qquad e^{At} \;=\; \frac{e^{-3t}}{3}\begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} \qquad y \;=\; \frac{e^{-3t}}{3}\begin{bmatrix} 1 \\ -2 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

[5] Solve the differential equation $y' = Ay$ where

$$A \;=\; \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}, \qquad y(0) \;=\; \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\lambda = -1, 2 \qquad e^{At} \;=\; \frac{e^{-t}}{3}\begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} + \frac{e^{2t}}{3}\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} \qquad y \;=\; \frac{e^{-t}}{3}\begin{bmatrix} -1 \\ 2 \end{bmatrix} + \frac{e^{2t}}{3}\begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

[6] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} -3 & 2 \\ -3 & 2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\lambda = -1, 0 \qquad e^{At} = e^{-t} \begin{bmatrix} 3 & -2 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} -2 & 2 \\ -3 & 3 \end{bmatrix} \qquad y = e^{-t} \begin{bmatrix} -1 \\ -1 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

[7] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} -2 & -1 \\ -2 & -1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\lambda = -3, 0 \qquad e^{At} = \frac{e^{-3t}}{3} \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} \qquad y = \frac{e^{-3t}}{3} \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

[8] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} -1 & -2 \\ 1 & 2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\lambda = 0, 1 \qquad e^{At} = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix} + e^{t} \begin{bmatrix} -1 & -2 \\ 1 & 2 \end{bmatrix} \qquad y = \begin{bmatrix} 2 \\ -1 \end{bmatrix} + e^{t} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

2×2 Exercise Set D (repeated roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find A^n where A is the matrix

$$A = \begin{bmatrix} 1 & 3 \\ -3 & -5 \end{bmatrix}$$

$$\lambda = -2, -2 \qquad A^n = (-2)^n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + n\,(-2)^{n-1} \begin{bmatrix} 3 & 3 \\ -3 & -3 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

$$\lambda = 0, 0 \qquad A^n = 0^n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + n\,0^{n-1} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

[3] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$$

$$\lambda = 1, 1 \qquad A^n = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + n \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & -1 \\ 4 & 6 \end{bmatrix}$$

$$\lambda = 4, 4 \qquad A^n = 4^n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + n\,4^{n-1} \begin{bmatrix} -2 & -1 \\ 4 & 2 \end{bmatrix}$$

[5] Find A^n where A is the matrix

$$A = \begin{bmatrix} 6 & 5 \\ -5 & -4 \end{bmatrix}$$

$$\lambda = 1, 1 \qquad A^n = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + n \begin{bmatrix} 5 & 5 \\ -5 & -5 \end{bmatrix}$$

[6] Find A^n where A is the matrix

$$A = \begin{bmatrix} 6 & 1 \\ -1 & 4 \end{bmatrix}$$

$$\lambda = 5, 5 \qquad A^n \;=\; 5^n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \;+\; n\, 5^{n-1} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

[7] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix}$$

$$\lambda = -1, -1 \qquad A^n \;=\; (-1)^n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \;+\; n\,(-1)^{n-1} \begin{bmatrix} 5 & -5 \\ 5 & -5 \end{bmatrix}$$

[8] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} -5 & -4 \\ 1 & -1 \end{bmatrix}$$

$$\lambda = -3, -3 \qquad A^n \;=\; (-3)^n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \;+\; n\,(-3)^{n-1} \begin{bmatrix} -2 & -4 \\ 1 & 2 \end{bmatrix}$$

2×2 Exercise Set E (repeated roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 5 & -4 \\ 4 & -3 \end{bmatrix}$$

$$\lambda = 1, 1 \qquad e^{At} = e^{t} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + te^{t} \begin{bmatrix} 4 & -4 \\ 4 & -4 \end{bmatrix}$$

[2] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -5 & -4 \\ 4 & 3 \end{bmatrix}$$

$$\lambda = -1, -1 \qquad e^{At} = e^{-t} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + te^{-t} \begin{bmatrix} -4 & -4 \\ 4 & 4 \end{bmatrix}$$

[3] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -3 & -1 \\ 1 & -1 \end{bmatrix}$$

$$\lambda = -2, -2 \qquad e^{At} = e^{-2t} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + te^{-2t} \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 5 & 1 \\ -1 & 3 \end{bmatrix}$$

$$\lambda = 4, 4 \qquad e^{At} = e^{4t} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + te^{4t} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

$$\lambda = 0, 0 \qquad e^{At} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + t \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -4 & 3 \\ -3 & 2 \end{bmatrix}$$

$$\lambda = -1, -1 \qquad e^{At} \;=\; e^{-t} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \;+\; te^{-t} \begin{bmatrix} -3 & 3 \\ -3 & 3 \end{bmatrix}$$

[7] Find e^{At} where A is the matrix

$$A \;=\; \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}$$

$$\lambda = 1, 1 \qquad e^{At} \;=\; e^{t} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \;+\; te^{t} \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$$

[8] Find e^{At} where A is the matrix

$$A \;=\; \begin{bmatrix} -1 & 2 \\ -2 & -5 \end{bmatrix}$$

$$\lambda = -3, -3 \qquad e^{At} \;=\; e^{-3t} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \;+\; te^{-3t} \begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix}$$

2×2 Exercise Set F (repeated roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 5 & 4 \\ -4 & -3 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda = 1, 1 \qquad e^{At} = e^{t}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + te^{t}\begin{bmatrix} 4 & 4 \\ -4 & -4 \end{bmatrix} \qquad y = e^{t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + te^{t}\begin{bmatrix} 8 \\ -8 \end{bmatrix}$$

[2] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & -4 \\ 1 & 6 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\lambda = 4, 4 \qquad e^{At} = e^{4t}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + te^{4t}\begin{bmatrix} -2 & -4 \\ 1 & 2 \end{bmatrix} \qquad y = e^{4t}\begin{bmatrix} 2 \\ 1 \end{bmatrix} + te^{4t}\begin{bmatrix} -8 \\ 4 \end{bmatrix}$$

[3] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} -1 & 4 \\ -1 & -5 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\lambda = -3, -3 \qquad e^{At} = e^{-3t}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + te^{-3t}\begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix} \qquad y = e^{-3t}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + te^{-3t}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

[4] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\lambda = 0, 0 \qquad e^{At} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + t\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \qquad y = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

[5] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} -2 & -1 \\ 4 & 2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\lambda = 0, 0 \qquad e^{At} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + t\begin{bmatrix} -2 & -1 \\ 4 & 2 \end{bmatrix} \qquad y = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + t\begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

[6] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\lambda = 0, 0 \qquad e^{At} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + t\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} \qquad y = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + t\begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

[7] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} -5 & -4 \\ 4 & 3 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\lambda = -1, -1 \qquad e^{At} = e^{-t}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + te^{-t}\begin{bmatrix} -4 & -4 \\ 4 & 4 \end{bmatrix} \qquad y = e^{-t}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + te^{-t}\begin{bmatrix} -12 \\ 12 \end{bmatrix}$$

[8] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} -4 & -3 \\ 3 & 2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\lambda = -1, -1 \qquad e^{At} = e^{-t}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + te^{-t}\begin{bmatrix} -3 & -3 \\ 3 & 3 \end{bmatrix} \qquad y = e^{-t}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + te^{-t}\begin{bmatrix} -9 \\ 9 \end{bmatrix}$$

2×2 Exercise Set G (symmetric matrices)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} -3 & -2 \\ -2 & -3 \end{bmatrix}$$

$$\lambda = -5, -1 \qquad A^n \;=\; \frac{(-5)^n}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \frac{(-1)^n}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} -1 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\lambda = -2, 3 \qquad A^n \;=\; \frac{(-2)^n}{5}\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix} + \frac{3^n}{5}\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

[3] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$$

$$\lambda = 0, 5 \qquad A^n \;=\; \frac{0^n}{5}\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} + \frac{5^n}{5}\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} -4 & -2 \\ -2 & -1 \end{bmatrix}$$

$$\lambda = -5, 0 \qquad A^n \;=\; \frac{(-5)^n}{5}\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} + \frac{0^n}{5}\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$$

[5] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}$$

$$\lambda = -4, -2 \qquad A^n \;=\; \frac{(-4)^n}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{(-2)^n}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

[6] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix}$$

$$\lambda = -3, -1 \qquad A^n = \frac{(-3)^n}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \frac{(-1)^n}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

[7] Find A^n where A is the matrix

$$A = \begin{bmatrix} -4 & 1 \\ 1 & -4 \end{bmatrix}$$

$$\lambda = -5, -3 \qquad A^n = \frac{(-5)^n}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{(-3)^n}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

[8] Find A^n where A is the matrix

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}$$

$$\lambda = -1, 4 \qquad A^n = \frac{(-1)^n}{5} \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} + \frac{4^n}{5} \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$$

2×2 Exercise Set H (symmetric matrices)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -3 & -2 \\ -2 & 0 \end{bmatrix}$$

$$\lambda = -4, 1 \qquad e^{At} = \frac{e^{-4t}}{5} \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} + \frac{e^{t}}{5} \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$$

[2] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$$

$$\lambda = -1, 4 \qquad e^{At} = \frac{e^{-t}}{5} \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix} + \frac{e^{4t}}{5} \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

[3] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$$

$$\lambda = -3, 1 \qquad e^{At} = \frac{e^{-3t}}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{e^{t}}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix}$$

$$\lambda = -5, 0 \qquad e^{At} = \frac{e^{-5t}}{5} \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} + \frac{1}{5} \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

$$\lambda = 2, 4 \qquad e^{At} = \frac{e^{2t}}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{e^{4t}}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -1 & -2 \\ -2 & 2 \end{bmatrix}$$

$$\lambda = -2, 3 \qquad e^{At} = \frac{e^{-2t}}{5}\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} + \frac{e^{3t}}{5}\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$$

[7] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -5 & -2 \\ -2 & -2 \end{bmatrix}$$

$$\lambda = -6, -1 \qquad e^{At} = \frac{e^{-6t}}{5}\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} + \frac{e^{-t}}{5}\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$$

[8] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -2 & -2 \\ -2 & -5 \end{bmatrix}$$

$$\lambda = -6, -1 \qquad e^{At} = \frac{e^{-6t}}{5}\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} + \frac{e^{-t}}{5}\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$$

2×2 Exercise Set I (quadratic forms)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Express the quadratic form

$$3x^2 - 2xy + 3y^2$$

as a sum of squares of othogonal linear forms.

$$\lambda = 2, 4 \qquad A = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + 2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$3x^2 - 2xy + 3y^2 = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = (x + y)^2 + 2\,(x - y)^2$$

[2] Express the quadratic form

$$-3x^2 + 2xy - 3y^2$$

as a sum of squares of othogonal linear forms.

$$\lambda = -4, -2 \qquad A = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} = -2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$-3x^2 + 2xy - 3y^2 = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = -2\,(x - y)^2 - (x + y)^2$$

[3] Express the quadratic form

$$-x^2 - 4xy - y^2$$

as a sum of squares of othogonal linear forms.

$$\lambda = -3, 1 \qquad A = \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix} = -\frac{3}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$-x^2 - 4xy - y^2 = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{3}{2}\,(x + y)^2 + \frac{1}{2}\,(x - y)^2$$

[4] Express the quadratic form

$$2x^2 - 4xy + 5y^2$$

as a sum of squares of othogonal linear forms.

$$\lambda = 1, 6 \qquad A = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} + \frac{6}{5} \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$$

$$2x^2 - 4xy + 5y^2 = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\,(2x + y)^2 + \frac{6}{5}\,(x - 2y)^2$$

[5] Express the quadratic form

$$2x^2 + 4xy - y^2$$

as a sum of squares of othogonal linear forms.

$$\lambda = -2, 3 \qquad A = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix} = -\frac{2}{5}\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} + \frac{3}{5}\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$$

$$2x^2 + 4xy - y^2 = \begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{2}{5}(x - 2y)^2 + \frac{3}{5}(2x + y)^2$$

[6] Express the quadratic form

$$3x^2 + 2xy + 3y^2$$

as a sum of squares of othogonal linear forms.

$$\lambda = 2, 4 \qquad A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$3x^2 + 2xy + 3y^2 = \begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = (x - y)^2 + 2(x + y)^2$$

[7] Express the quadratic form

$$-2x^2 + 4xy + y^2$$

as a sum of squares of othogonal linear forms.

$$\lambda = -3, 2 \qquad A = \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix} = -\frac{3}{5}\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix} + \frac{2}{5}\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

$$-2x^2 + 4xy + y^2 = \begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{3}{5}(2x - y)^2 + \frac{2}{5}(x + 2y)^2$$

[8] Express the quadratic form

$$-x^2 - 8xy - y^2$$

as a sum of squares of othogonal linear forms.

$$\lambda = -5, 3 \qquad A = \begin{bmatrix} -1 & -4 \\ -4 & -1 \end{bmatrix} = -\frac{5}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \frac{3}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$-x^2 - 8xy - y^2 = \begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} -1 & -4 \\ -4 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{5}{2}(x + y)^2 + \frac{3}{2}(x - y)^2$$

2×2 Exercise Set J (recurrence relations)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = -5\,f(n-1) - 4\,f(n-2)$$

$$\begin{bmatrix} f(n+1) \\ f(n) \end{bmatrix} = \begin{bmatrix} -5 & -4 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} b \\ a \end{bmatrix} = \frac{(-4)^n}{3} \begin{bmatrix} 4 & 4 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} + \frac{(-1)^n}{3} \begin{bmatrix} -1 & -4 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix}$$

$$f(n) = \frac{(-4)^n}{3}(-b - a) + \frac{(-1)^n}{3}(b + 4a)$$

[2] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = 6\,f(n-1) - 5\,f(n-2)$$

$$\begin{bmatrix} f(n+1) \\ f(n) \end{bmatrix} = \begin{bmatrix} 6 & -5 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} b \\ a \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -1 & 5 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} + \frac{5^n}{4} \begin{bmatrix} 5 & -5 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix}$$

$$f(n) = \frac{1}{4}(-b + 5a) + \frac{5^n}{4}(b - a)$$

[3] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = -6\,f(n-1) - 8\,f(n-2)$$

$$\begin{bmatrix} f(n+1) \\ f(n) \end{bmatrix} = \begin{bmatrix} -6 & -8 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} b \\ a \end{bmatrix} = \frac{(-4)^n}{2} \begin{bmatrix} 4 & 8 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} + \frac{(-2)^n}{2} \begin{bmatrix} -2 & -8 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix}$$

$$f(n) = \frac{(-4)^n}{2}(-b - 2a) + \frac{(-2)^n}{2}(b + 4a)$$

[4] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = -4\,f(n-1) + 5\,f(n-2)$$

$$\begin{bmatrix} f(n+1) \\ f(n) \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} b \\ a \end{bmatrix} = \frac{(-5)^n}{6} \begin{bmatrix} 5 & -5 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} + \frac{1}{6} \begin{bmatrix} 1 & 5 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix}$$

$$f(n) = \frac{(-5)^n}{6}(-b + a) + \frac{1}{6}(b + 5a)$$

[5] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = -\, f(n-1) \; + \; 6\, f(n-2)$$

$$\begin{bmatrix} f(n+1) \\ f(n) \end{bmatrix} = \begin{bmatrix} -1 & 6 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} b \\ a \end{bmatrix} = \frac{(-3)^n}{5} \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} + \frac{2^n}{5} \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix}$$

$$f(n) = \frac{(-3)^n}{5} (\, -b \; + \; 2a) \; + \; \frac{2^n}{5} (b \; + \; 3a)$$

[6] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = -\,6\, f(n-1) \; + \; 7\, f(n-2)$$

$$\begin{bmatrix} f(n+1) \\ f(n) \end{bmatrix} = \begin{bmatrix} -6 & 7 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} b \\ a \end{bmatrix} = \frac{(-7)^n}{8} \begin{bmatrix} 7 & -7 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} + \frac{1}{8} \begin{bmatrix} 1 & 7 \\ 1 & 7 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix}$$

$$f(n) = \frac{(-7)^n}{8} (\, -b \; + \; a) \; + \; \frac{1}{8} (b \; + \; 7a)$$

[7] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = -\,6\, f(n-1) \; - \; 5\, f(n-2)$$

$$\begin{bmatrix} f(n+1) \\ f(n) \end{bmatrix} = \begin{bmatrix} -6 & -5 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} b \\ a \end{bmatrix} = \frac{(-5)^n}{4} \begin{bmatrix} 5 & 5 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} + \frac{(-1)^n}{4} \begin{bmatrix} -1 & -5 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix}$$

$$f(n) = \frac{(-5)^n}{4} (\, -b \; - \; a) \; + \; \frac{(-1)^n}{4} (b \; + \; 5a)$$

[8] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = f(n-1) \; + \; 6\, f(n-2)$$

$$\begin{bmatrix} f(n+1) \\ f(n) \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} b \\ a \end{bmatrix} = \frac{(-2)^n}{5} \begin{bmatrix} 2 & -6 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} + \frac{3^n}{5} \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix}$$

$$f(n) = \frac{(-2)^n}{5} (\, -b \; + \; 3a) \; + \; \frac{3^n}{5} (b \; + \; 2a)$$

3×3 Exercise Set A (distinct roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$\lambda = 0, 2, 3 \qquad A^n = \frac{0^n}{6} \begin{bmatrix} 0 & 0 & 0 \\ -3 & 4 & -2 \\ 3 & -4 & 2 \end{bmatrix} + \frac{2^n}{2} \begin{bmatrix} 2 & 0 & 0 \\ -1 & 0 & 0 \\ -5 & 0 & 0 \end{bmatrix} + \frac{3^n}{3} \begin{bmatrix} 0 & 0 & 0 \\ 3 & 1 & 1 \\ 6 & 2 & 2 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$\lambda = -1, 1, 3 \qquad A^n = \frac{(-1)^n}{2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix} + \frac{3^n}{2} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

[3] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\lambda = 0, 2, 3 \qquad A^n = \frac{0^n}{3} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & -2 \\ 0 & -1 & 1 \end{bmatrix} + 2^n \begin{bmatrix} 1 & -1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{3^n}{3} \begin{bmatrix} 0 & 3 & 6 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

$$\lambda = -1, 1, 3 \qquad A^n = \frac{(-1)^n}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 2 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{3^n}{2} \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

[5] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

$$\lambda = 0, 1, 3 \qquad A^n \;=\; \frac{0^n}{3}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -2 & 0 & 2 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ -2 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix} + \frac{3^n}{6}\begin{bmatrix} 4 & 0 & 2 \\ 6 & 0 & 3 \\ 4 & 0 & 2 \end{bmatrix}$$

[6] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\lambda = 0, 2, 3 \qquad A^n \;=\; \frac{0^n}{6}\begin{bmatrix} 4 & -4 & 2 \\ -2 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix} + \frac{2^n}{2}\begin{bmatrix} 0 & 0 & -4 \\ 0 & 0 & -3 \\ 0 & 0 & 2 \end{bmatrix} + \frac{3^n}{3}\begin{bmatrix} 1 & 2 & 5 \\ 1 & 2 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

[7] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 2 & 2 & 2 \end{bmatrix}$$

$$\lambda = 0, 1, 3 \qquad A^n \;=\; \frac{0^n}{3}\begin{bmatrix} 2 & 0 & -1 \\ 0 & 0 & 0 \\ -2 & 0 & 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & -2 & 0 \end{bmatrix} + \frac{3^n}{6}\begin{bmatrix} 2 & 3 & 2 \\ 0 & 0 & 0 \\ 4 & 6 & 4 \end{bmatrix}$$

[8] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 2 \\ 2 & 1 & 1 \end{bmatrix}$$

$$\lambda = 0, 2, 3 \qquad A^n \;=\; \frac{0^n}{3}\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & -2 \\ -1 & -1 & 2 \end{bmatrix} + 2^n\begin{bmatrix} 1 & 0 & 0 \\ -3 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} + \frac{3^n}{3}\begin{bmatrix} 0 & 0 & 0 \\ 8 & 2 & 2 \\ 4 & 1 & 1 \end{bmatrix}$$

3×3 Exercise Set B (distinct roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\lambda = 0, 2, 3 \qquad e^{At} = \frac{1}{6}\begin{bmatrix} 2 & -2 & 1 \\ -4 & 4 & -2 \\ 0 & 0 & 0 \end{bmatrix} + \frac{e^{2t}}{2}\begin{bmatrix} 0 & 0 & -3 \\ 0 & 0 & -2 \\ 0 & 0 & 2 \end{bmatrix} + \frac{e^{3t}}{3}\begin{bmatrix} 2 & 1 & 4 \\ 2 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

[2] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\lambda = -1, 1, 3 \qquad e^{At} = \frac{e^{-t}}{8}\begin{bmatrix} 1 & 1 & -3 \\ 1 & 1 & -3 \\ -2 & -2 & 6 \end{bmatrix} + \frac{e^{t}}{4}\begin{bmatrix} 1 & -3 & -1 \\ -1 & 3 & 1 \\ 0 & 0 & 0 \end{bmatrix} + \frac{e^{3t}}{8}\begin{bmatrix} 5 & 5 & 5 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$$

[3] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\lambda = -1, 1, 3 \qquad e^{At} = \frac{e^{-t}}{8}\begin{bmatrix} 3 & -1 & -5 \\ 0 & 0 & 0 \\ -3 & 1 & 5 \end{bmatrix} + \frac{e^{t}}{4}\begin{bmatrix} 1 & -1 & 1 \\ -2 & 2 & -2 \\ 1 & -1 & 1 \end{bmatrix} + \frac{e^{3t}}{8}\begin{bmatrix} 3 & 3 & 3 \\ 4 & 4 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\lambda = -1, 1, 3 \qquad e^{At} = \frac{e^{-t}}{8}\begin{bmatrix} 6 & -3 & -3 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix} + \frac{e^{t}}{4}\begin{bmatrix} 0 & 1 & -1 \\ 0 & 3 & -3 \\ 0 & -1 & 1 \end{bmatrix} + \frac{e^{3t}}{8}\begin{bmatrix} 2 & 1 & 5 \\ 2 & 1 & 5 \\ 2 & 1 & 5 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\lambda = 0, 1, 3 \qquad e^{At} \;=\; \frac{1}{3}\begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & -1 \\ 2 & -2 & 2 \end{bmatrix} + \frac{e^t}{2}\begin{bmatrix} 2 & 0 & 0 \\ -1 & 0 & 0 \\ -3 & 0 & 0 \end{bmatrix} + \frac{e^{3t}}{6}\begin{bmatrix} 0 & 0 & 0 \\ 5 & 4 & 2 \\ 5 & 4 & 2 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A \;=\; \begin{bmatrix} 2 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$\lambda = 0, 1, 3 \qquad e^{At} \;=\; \frac{1}{3}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -2 & 0 & 2 \end{bmatrix} + e^t\begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix} + \frac{e^{3t}}{3}\begin{bmatrix} 2 & 3 & 1 \\ 0 & 0 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

[7] Find e^{At} where A is the matrix

$$A \;=\; \begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\lambda = 1, 2, 3 \qquad e^{At} \;=\; \frac{e^t}{2}\begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + e^{2t}\begin{bmatrix} 1 & -1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{e^{3t}}{2}\begin{bmatrix} 0 & 3 & 3 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

[8] Find e^{At} where A is the matrix

$$A \;=\; \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\lambda = -1, 1, 3 \qquad e^{At} \;=\; \frac{e^{-t}}{4}\begin{bmatrix} 1 & -3 & 1 \\ -1 & 3 & -1 \\ 0 & 0 & 0 \end{bmatrix} + \frac{e^t}{2}\begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 0 \\ -1 & -1 & 1 \end{bmatrix} + \frac{e^{3t}}{4}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$$

3 × 3 Exercise Set C (distinct roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda = -1, 1, 3 \qquad e^{At} = \frac{e^{-t}}{8} \begin{bmatrix} 2 & -4 & 4 \\ -2 & 4 & -4 \\ 1 & -2 & 2 \end{bmatrix} + \frac{e^{t}}{4} \begin{bmatrix} 2 & 0 & -4 \\ 0 & 0 & 0 \\ -1 & 0 & 2 \end{bmatrix} + \frac{e^{3t}}{8} \begin{bmatrix} 2 & 4 & 4 \\ 2 & 4 & 4 \\ 1 & 2 & 2 \end{bmatrix}$$

$$y = \frac{e^{-t}}{8} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \frac{e^{t}}{4} \begin{bmatrix} -4 \\ 0 \\ 2 \end{bmatrix} + \frac{e^{3t}}{8} \begin{bmatrix} 8 \\ 8 \\ 4 \end{bmatrix}$$

[2] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 0 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda = 0, 2, 3 \qquad e^{At} = \frac{1}{3} \begin{bmatrix} 1 & 1 & -2 \\ 0 & 0 & 0 \\ -1 & -1 & 2 \end{bmatrix} + e^{2t} \begin{bmatrix} 0 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix} + \frac{e^{3t}}{3} \begin{bmatrix} 2 & 8 & 2 \\ 0 & 0 & 0 \\ 1 & 4 & 1 \end{bmatrix}$$

$$y = \frac{1}{3} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + e^{2t} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \frac{e^{3t}}{3} \begin{bmatrix} 6 \\ 0 \\ 3 \end{bmatrix}$$

[3] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \\ 0 & 2 & 2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda = 0, 2, 4 \qquad e^{At} = \frac{1}{8} \begin{bmatrix} 2 & -2 & 1 \\ -4 & 4 & -2 \\ 4 & -4 & 2 \end{bmatrix} + \frac{e^{2t}}{4} \begin{bmatrix} 2 & 0 & -1 \\ 0 & 0 & 0 \\ -4 & 0 & 2 \end{bmatrix} + \frac{e^{4t}}{8} \begin{bmatrix} 2 & 2 & 1 \\ 4 & 4 & 2 \\ 4 & 4 & 2 \end{bmatrix}$$

$$y = \frac{1}{8} \begin{bmatrix} 5 \\ -10 \\ 10 \end{bmatrix} + \frac{e^{2t}}{4} \begin{bmatrix} 3 \\ 0 \\ -6 \end{bmatrix} + \frac{e^{4t}}{8} \begin{bmatrix} 5 \\ 10 \\ 10 \end{bmatrix}$$

[4] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 1 \\ 2 & 2 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda = 0, 2, 3 \qquad e^{At} = \frac{1}{3}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -2 & 2 \end{bmatrix} + e^{2t}\begin{bmatrix} 1 & 0 & 0 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} + \frac{e^{3t}}{3}\begin{bmatrix} 0 & 0 & 0 \\ 6 & 2 & 1 \\ 6 & 2 & 1 \end{bmatrix}$$

$$y = \frac{1}{3}\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} + e^{2t}\begin{bmatrix} 2 \\ -4 \\ -4 \end{bmatrix} + \frac{e^{3t}}{3}\begin{bmatrix} 0 \\ 13 \\ 13 \end{bmatrix}$$

[5] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda = 0, 1, 3 \qquad e^{At} = \frac{1}{3}\begin{bmatrix} 1 & -2 & 0 \\ -1 & 2 & 0 \\ -1 & 2 & 0 \end{bmatrix} + \frac{e^{t}}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & -3 & 2 \end{bmatrix} + \frac{e^{3t}}{6}\begin{bmatrix} 4 & 4 & 0 \\ 2 & 2 & 0 \\ 5 & 5 & 0 \end{bmatrix}$$

$$y = \frac{1}{3}\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + \frac{e^{t}}{2}\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} + \frac{e^{3t}}{6}\begin{bmatrix} 8 \\ 4 \\ 10 \end{bmatrix}$$

[6] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda = 0, 1, 3 \qquad e^{At} = \frac{1}{3}\begin{bmatrix} 1 & -2 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{e^{t}}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & -1 & 2 \end{bmatrix} + \frac{e^{3t}}{6}\begin{bmatrix} 4 & 4 & 0 \\ 2 & 2 & 0 \\ 3 & 3 & 0 \end{bmatrix}$$

$$y = \frac{1}{3}\begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix} + \frac{e^{t}}{2}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{e^{3t}}{6}\begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$$

[7] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda = 0, 1, 3 \qquad e^{At} = \frac{1}{3}\begin{bmatrix} -1 & -1 & 1 \\ 2 & 2 & -2 \\ -2 & -2 & 2 \end{bmatrix} + \frac{e^t}{2}\begin{bmatrix} 2 & 0 & -1 \\ -2 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + \frac{e^{3t}}{6}\begin{bmatrix} 2 & 2 & 1 \\ 2 & 2 & 1 \\ 4 & 4 & 2 \end{bmatrix}$$

$$y = \frac{1}{3}\begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix} + \frac{e^t}{2}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \frac{e^{3t}}{6}\begin{bmatrix} 5 \\ 5 \\ 10 \end{bmatrix}$$

[8] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda = 0, 1, 3 \qquad e^{At} = \frac{1}{3}\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ -2 & 0 & 2 \end{bmatrix} + \frac{e^t}{2}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \end{bmatrix} + \frac{e^{3t}}{6}\begin{bmatrix} 1 & 3 & 2 \\ 1 & 3 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

$$y = \frac{1}{3}\begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} + \frac{e^t}{2}\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} + \frac{e^{3t}}{6}\begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$$

3×3 Exercise Set D (repeated roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\lambda = 3, 1, 1 \qquad A^n = \frac{3^n}{4} \begin{bmatrix} 2 & 2 & 2 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 2 & -2 & -2 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} + \frac{n}{2} \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

$$\lambda = 4, 1, 1 \qquad A^n = \frac{4^n}{9} \begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 4 & 4 & 4 \end{bmatrix} + \frac{1}{9} \begin{bmatrix} 6 & -3 & -3 \\ -2 & 7 & -2 \\ -4 & -4 & 5 \end{bmatrix} + \frac{n}{3} \begin{bmatrix} 0 & 0 & 0 \\ -2 & 1 & 1 \\ 2 & -1 & -1 \end{bmatrix}$$

[3] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\lambda = 3, 1, 1 \qquad A^n = \frac{3^n}{4} \begin{bmatrix} 2 & 0 & 2 \\ 3 & 0 & 3 \\ 2 & 0 & 2 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 2 & 0 & -2 \\ -3 & 4 & -3 \\ -2 & 0 & 2 \end{bmatrix} + \frac{n}{2} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\lambda = 4, 1, 1 \qquad A^n = \frac{4^n}{9} \begin{bmatrix} 4 & 3 & 2 \\ 4 & 3 & 2 \\ 4 & 3 & 2 \end{bmatrix} + \frac{1}{9} \begin{bmatrix} 5 & -3 & -2 \\ -4 & 6 & -2 \\ -4 & -3 & 7 \end{bmatrix} + \frac{n}{3} \begin{bmatrix} -1 & 0 & 1 \\ 2 & 0 & -2 \\ -1 & 0 & 1 \end{bmatrix}$$

[5] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\lambda = 4, 1, 1 \qquad A^n = \frac{4^n}{9}\begin{bmatrix} 3 & 6 & 6 \\ 2 & 4 & 4 \\ 1 & 2 & 2 \end{bmatrix} + \frac{1}{9}\begin{bmatrix} 6 & -6 & -6 \\ -2 & 5 & -4 \\ -1 & -2 & 7 \end{bmatrix} + \frac{n}{3}\begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

[6] Find A^n where A is the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\lambda = 3, 1, 1 \qquad A^n = \frac{3^n}{2}\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} + n\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

[7] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix}$$

$$\lambda = 4, 1, 1 \qquad A^n = \frac{4^n}{9}\begin{bmatrix} 4 & 4 & 2 \\ 2 & 2 & 1 \\ 6 & 6 & 3 \end{bmatrix} + \frac{1}{9}\begin{bmatrix} 5 & -4 & -2 \\ -2 & 7 & -1 \\ -6 & -6 & 6 \end{bmatrix} + \frac{n}{3}\begin{bmatrix} -1 & -1 & 1 \\ 1 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

[8] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\lambda = 3, 1, 1 \qquad A^n = \frac{3^n}{4}\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} + \frac{1}{4}\begin{bmatrix} 3 & -1 & -1 \\ -2 & 2 & -2 \\ -1 & -1 & 3 \end{bmatrix} + \frac{n}{2}\begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix}$$

3×3 Exercise Set E (repeated roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

$$\lambda = 3, 1, 1 \qquad e^{At} = \frac{e^{3t}}{4} \begin{bmatrix} 0 & 0 & 0 \\ 3 & 2 & 2 \\ 3 & 2 & 2 \end{bmatrix} + \frac{e^{t}}{4} \begin{bmatrix} 4 & 0 & 0 \\ -3 & 2 & -2 \\ -3 & -2 & 2 \end{bmatrix} + \frac{te^{t}}{2} \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

[2] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\lambda = 3, 1, 1 \qquad e^{At} = \frac{e^{3t}}{2} \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} + \frac{e^{t}}{2} \begin{bmatrix} 2 & -1 & -1 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + te^{t} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

[3] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

$$\lambda = 4, 1, 1 \qquad e^{At} = \frac{e^{4t}}{9} \begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 4 & 4 & 4 \end{bmatrix} + \frac{e^{t}}{9} \begin{bmatrix} 6 & -3 & -3 \\ -2 & 7 & -2 \\ -4 & -4 & 5 \end{bmatrix} + \frac{te^{t}}{3} \begin{bmatrix} 0 & 0 & 0 \\ -2 & 1 & 1 \\ 2 & -1 & -1 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\lambda = 3, 0, 0 \qquad e^{At} = \frac{e^{3t}}{9} \begin{bmatrix} 2 & 3 & 1 \\ 4 & 6 & 2 \\ 2 & 3 & 1 \end{bmatrix} + \frac{1}{9} \begin{bmatrix} 7 & -3 & -1 \\ -4 & 3 & -2 \\ -2 & -3 & 8 \end{bmatrix} + \frac{t}{3} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\lambda = 4, 1, 1 \qquad e^{At} = \frac{e^{4t}}{9}\begin{bmatrix} 3 & 4 & 2 \\ 3 & 4 & 2 \\ 3 & 4 & 2 \end{bmatrix} + \frac{e^{t}}{9}\begin{bmatrix} 6 & -4 & -2 \\ -3 & 5 & -2 \\ -3 & -4 & 7 \end{bmatrix} + \frac{te^{t}}{3}\begin{bmatrix} 0 & 2 & -2 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

$$\lambda = 0, 2, 2 \qquad e^{At} = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{e^{2t}}{2}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} + te^{2t}\begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

[7] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\lambda = 0, 2, 2 \qquad e^{At} = \frac{1}{4}\begin{bmatrix} 0 & 0 & 0 \\ -1 & 2 & -2 \\ 1 & -2 & 2 \end{bmatrix} + \frac{e^{2t}}{4}\begin{bmatrix} 4 & 0 & 0 \\ 1 & 2 & 2 \\ -1 & 2 & 2 \end{bmatrix} + \frac{te^{2t}}{2}\begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

[8] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\lambda = 3, 1, 1 \qquad e^{At} = \frac{e^{3t}}{4}\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} + \frac{e^{t}}{4}\begin{bmatrix} 3 & -1 & -1 \\ -2 & 2 & -2 \\ -1 & -1 & 3 \end{bmatrix} + \frac{te^{t}}{2}\begin{bmatrix} -1 & 1 & -1 \\ 0 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

3×3 Exercise Set F (repeated roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Solve the differential equation $y' = Ay$ where

$$A \;=\; \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \qquad y(0) \;=\; \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda = 3, 1, 1 \qquad e^{At} \;=\; \frac{e^{3t}}{2} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} + \frac{e^{t}}{2} \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix} + te^{t} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$y \;=\; \frac{e^{3t}}{2} \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} + \frac{e^{t}}{2} \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} + te^{t} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

[2] Solve the differential equation $y' = Ay$ where

$$A \;=\; \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}, \qquad y(0) \;=\; \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda = 3, 1, 1 \qquad e^{At} \;=\; \frac{e^{3t}}{4} \begin{bmatrix} 2 & 0 & 2 \\ 3 & 0 & 3 \\ 2 & 0 & 2 \end{bmatrix} + \frac{e^{t}}{4} \begin{bmatrix} 2 & 0 & -2 \\ -3 & 4 & -3 \\ -2 & 0 & 2 \end{bmatrix} + \frac{te^{t}}{2} \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$y \;=\; \frac{e^{3t}}{4} \begin{bmatrix} 4 \\ 6 \\ 4 \end{bmatrix} + \frac{e^{t}}{4} \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix} + \frac{te^{t}}{2} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

[3] Solve the differential equation $y' = Ay$ where

$$A \;=\; \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}, \qquad y(0) \;=\; \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda = 4, 1, 1 \qquad e^{At} \;=\; \frac{e^{4t}}{9} \begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 4 & 4 & 4 \end{bmatrix} + \frac{e^{t}}{9} \begin{bmatrix} 6 & -3 & -3 \\ -2 & 7 & -2 \\ -4 & -4 & 5 \end{bmatrix} + \frac{te^{t}}{3} \begin{bmatrix} 0 & 0 & 0 \\ -2 & 1 & 1 \\ 2 & -1 & -1 \end{bmatrix}$$

$$y \;=\; \frac{e^{4t}}{9} \begin{bmatrix} 9 \\ 6 \\ 12 \end{bmatrix} + \frac{e^{t}}{9} \begin{bmatrix} 9 \\ -6 \\ -3 \end{bmatrix} + \frac{te^{t}}{3} \begin{bmatrix} 0 \\ -3 \\ 3 \end{bmatrix}$$

[4] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda = 3, 0, 0 \qquad e^{At} = \frac{e^{3t}}{9}\begin{bmatrix} 3 & 3 & 3 \\ 4 & 4 & 4 \\ 2 & 2 & 2 \end{bmatrix} + \frac{1}{9}\begin{bmatrix} 6 & -3 & -3 \\ -4 & 5 & -4 \\ -2 & -2 & 7 \end{bmatrix} + \frac{t}{3}\begin{bmatrix} 0 & 0 & 0 \\ 2 & -1 & -1 \\ -2 & 1 & 1 \end{bmatrix}$$

$$y = \frac{e^{3t}}{9}\begin{bmatrix} 6 \\ 8 \\ 4 \end{bmatrix} + \frac{1}{9}\begin{bmatrix} -6 \\ 1 \\ 5 \end{bmatrix} + \frac{t}{3}\begin{bmatrix} 0 \\ -2 \\ 2 \end{bmatrix}$$

[5] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$\lambda = 4, 1, 1 \qquad e^{At} = \frac{e^{4t}}{9}\begin{bmatrix} 4 & 4 & 2 \\ 2 & 2 & 1 \\ 6 & 6 & 3 \end{bmatrix} + \frac{e^{t}}{9}\begin{bmatrix} 5 & -4 & -2 \\ -2 & 7 & -1 \\ -6 & -6 & 6 \end{bmatrix} + \frac{te^{t}}{3}\begin{bmatrix} -1 & -1 & 1 \\ 1 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$y = \frac{e^{4t}}{9}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} + \frac{e^{t}}{9}\begin{bmatrix} -8 \\ 5 \\ 6 \end{bmatrix} + \frac{te^{t}}{3}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

[6] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda = 3, 1, 1 \qquad e^{At} = \frac{e^{3t}}{4}\begin{bmatrix} 0 & 0 & 0 \\ 3 & 2 & 2 \\ 3 & 2 & 2 \end{bmatrix} + \frac{e^{t}}{4}\begin{bmatrix} 4 & 0 & 0 \\ -3 & 2 & -2 \\ -3 & -2 & 2 \end{bmatrix} + \frac{te^{t}}{2}\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$y = \frac{e^{3t}}{4}\begin{bmatrix} 0 \\ 7 \\ 7 \end{bmatrix} + \frac{e^{t}}{4}\begin{bmatrix} 4 \\ -3 \\ -3 \end{bmatrix} + \frac{te^{t}}{2}\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

[7] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda = 0, 2, 2 \qquad e^{At} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{e^{2t}}{2}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} + te^{2t}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$y = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \frac{e^{2t}}{2}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + te^{2t}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

[8] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda = 3, 0, 0 \qquad e^{At} = \frac{e^{3t}}{9}\begin{bmatrix} 2 & 4 & 3 \\ 2 & 4 & 3 \\ 2 & 4 & 3 \end{bmatrix} + \frac{1}{9}\begin{bmatrix} 7 & -4 & -3 \\ -2 & 5 & -3 \\ -2 & -4 & 6 \end{bmatrix} + \frac{t}{3}\begin{bmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ -2 & 2 & 0 \end{bmatrix}$$

$$y = \frac{e^{3t}}{9}\begin{bmatrix} 7 \\ 7 \\ 7 \end{bmatrix} + \frac{1}{9}\begin{bmatrix} -7 \\ 2 \\ 2 \end{bmatrix} + \frac{t}{3}\begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$

3×3 Exercise Set G (identical roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 3 & 3 \\ -2 & 3 & 1 \\ 2 & -1 & 1 \end{bmatrix}$$

$$\lambda = 2, 2, 2$$

$$A^n = 2^n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + n\, 2^{n-1} \begin{bmatrix} 0 & 3 & 3 \\ -2 & 1 & 1 \\ 2 & -1 & -1 \end{bmatrix} + \frac{n(n-1)\, 2^{n-2}}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -6 & -6 \\ 0 & 6 & 6 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 1 \\ 1 & -2 & -2 \end{bmatrix}$$

$$\lambda = -1, -1, -1$$

$$A^n = (-1)^n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + n\, (-1)^{n-1} \begin{bmatrix} 2 & -1 & 1 \\ 2 & -1 & 1 \\ 1 & -2 & -1 \end{bmatrix} + \frac{n(n-1)\, (-1)^{n-2}}{2} \begin{bmatrix} 3 & -3 & 0 \\ 3 & -3 & 0 \\ -3 & 3 & 0 \end{bmatrix}$$

[3] Find A^n where A is the matrix

$$A = \begin{bmatrix} 3 & 3 & -2 \\ -1 & -1 & -2 \\ 1 & 1 & -2 \end{bmatrix}$$

$$\lambda = 0, 0, 0$$

$$A^n = 0^n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + n\, 0^{n-1} \begin{bmatrix} 3 & 3 & -2 \\ -1 & -1 & -2 \\ 1 & 1 & -2 \end{bmatrix} + \frac{n(n-1)\, 0^{n-2}}{2} \begin{bmatrix} 4 & 4 & -8 \\ -4 & -4 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -1 \\ 1 & 3 & 1 \end{bmatrix}$$

$$\lambda = 0, 0, 0$$

$$A^n = 0^n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + n\,0^{n-1} \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -1 \\ 1 & 3 & 1 \end{bmatrix} + \frac{n(n-1)\,0^{n-2}}{2} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

[5] Find A^n where A is the matrix

$$A = \begin{bmatrix} 3 & 2 & 2 \\ -2 & -1 & -2 \\ 3 & 3 & 1 \end{bmatrix}$$

$$\lambda = 1, 1, 1$$

$$A^n = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + n \begin{bmatrix} 2 & 2 & 2 \\ -2 & -2 & -2 \\ 3 & 3 & 0 \end{bmatrix} + \frac{n(n-1)}{2} \begin{bmatrix} 6 & 6 & 0 \\ -6 & -6 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

[6] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & -1 & 1 \\ -1 & -2 & -1 \end{bmatrix}$$

$$\lambda = 0, 0, 0$$

$$A^n = 0^n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + n\,0^{n-1} \begin{bmatrix} 2 & 1 & -1 \\ -2 & -1 & 1 \\ -1 & -2 & -1 \end{bmatrix} + \frac{n(n-1)\,0^{n-2}}{2} \begin{bmatrix} 3 & 3 & 0 \\ -3 & -3 & 0 \\ 3 & 3 & 0 \end{bmatrix}$$

[7] Find A^n where A is the matrix

$$A = \begin{bmatrix} -1 & 2 & 1 \\ -1 & 2 & 2 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\lambda = 1, 1, 1$$

$$A^n = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + n \begin{bmatrix} -2 & 2 & 1 \\ -1 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \frac{n(n-1)}{2} \begin{bmatrix} 3 & -3 & 3 \\ 3 & -3 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

[8] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & -2 & -1 \\ 2 & 2 & -1 \\ -2 & -2 & 2 \end{bmatrix}$$

$$\lambda = 2, 2, 2$$

$$A^n \;=\; 2^n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \;+\; n\,2^{n-1} \begin{bmatrix} 0 & -2 & -1 \\ 2 & 0 & -1 \\ -2 & -2 & 0 \end{bmatrix} \;+\; \frac{n(n-1)\,2^{n-2}}{2} \begin{bmatrix} -2 & 2 & 2 \\ 2 & -2 & -2 \\ -4 & 4 & 4 \end{bmatrix}$$

3×3 Exercise Set H (identical roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & 1 & -1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\lambda = 2, 2, 2$$

$$e^{At} = e^{2t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^{2t} \begin{bmatrix} 0 & 3 & 3 \\ 1 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix} + \frac{t^2 e^{2t}}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 3 \\ 0 & -3 & -3 \end{bmatrix}$$

[2] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -2 & 2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\lambda = 0, 0, 0$$

$$e^{At} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + t \begin{bmatrix} -2 & 2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & 1 \end{bmatrix} + \frac{t^2}{2} \begin{bmatrix} 4 & -4 & 4 \\ 4 & -4 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

[3] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -1 & 3 & 2 \\ 2 & 1 & -2 \\ -2 & 3 & 3 \end{bmatrix}$$

$$\lambda = 1, 1, 1$$

$$e^{At} = e^{t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^{t} \begin{bmatrix} -2 & 3 & 2 \\ 2 & 0 & -2 \\ -2 & 3 & 2 \end{bmatrix} + \frac{t^2 e^{t}}{2} \begin{bmatrix} 6 & 0 & -6 \\ 0 & 0 & 0 \\ 6 & 0 & -6 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -2 & 2 & 2 \\ 1 & -2 & 1 \\ -2 & 2 & -2 \end{bmatrix}$$

$$\lambda = -2, -2, -2$$

$$e^{At} = e^{-2t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^{-2t} \begin{bmatrix} 0 & 2 & 2 \\ 1 & 0 & 1 \\ -2 & 2 & 0 \end{bmatrix} + \frac{t^2e^{-2t}}{2} \begin{bmatrix} -2 & 4 & 2 \\ -2 & 4 & 2 \\ 2 & -4 & -2 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -2 & -1 & 1 \\ -1 & -2 & 3 \\ -1 & -1 & 1 \end{bmatrix}$$

$$\lambda = -1, -1, -1$$

$$e^{At} = e^{-t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^{-t} \begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & 3 \\ -1 & -1 & 2 \end{bmatrix} + \frac{t^2e^{-t}}{2} \begin{bmatrix} 1 & 1 & -2 \\ -1 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 1 & 2 & -2 \\ 1 & 2 & 2 \end{bmatrix}$$

$$\lambda = 2, 2, 2$$

$$e^{At} = e^{2t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^{2t} \begin{bmatrix} 0 & 2 & 2 \\ 1 & 0 & -2 \\ 1 & 2 & 0 \end{bmatrix} + \frac{t^2e^{2t}}{2} \begin{bmatrix} 4 & 4 & -4 \\ -2 & -2 & 2 \\ 2 & 2 & -2 \end{bmatrix}$$

[7] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 3 & 2 & 2 \\ -2 & -1 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

$$\lambda = 1, 1, 1$$

$$e^{At} = e^{t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^{t} \begin{bmatrix} 2 & 2 & 2 \\ -2 & -2 & -2 \\ -2 & -2 & 0 \end{bmatrix} + \frac{t^2e^{t}}{2} \begin{bmatrix} -4 & -4 & 0 \\ 4 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

[8] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & -2 & -2 \\ -1 & 2 & 1 \\ 2 & 2 & 2 \end{bmatrix}$$

$$\lambda = 2, 2, 2$$

$$e^{At} = e^{2t}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^{2t}\begin{bmatrix} 0 & -2 & -2 \\ -1 & 0 & 1 \\ 2 & 2 & 0 \end{bmatrix} + \frac{t^2 e^{2t}}{2}\begin{bmatrix} -2 & -4 & -2 \\ 2 & 4 & 2 \\ -2 & -4 & -2 \end{bmatrix}$$

3×3 Exercise Set I (identical roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} -1 & 3 & 1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda = 1, 1, 1$$

$$e^{At} = e^{t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^{t} \begin{bmatrix} -2 & 3 & 1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix} + \frac{t^2 e^{t}}{2} \begin{bmatrix} 0 & -2 & 2 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$y = e^{t} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + te^{t} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \frac{t^2 e^{t}}{2} \begin{bmatrix} -2 \\ -1 \\ -1 \end{bmatrix}$$

[2] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & -2 & -2 \\ 2 & 2 & -2 \\ -1 & -1 & 2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda = 2, 2, 2$$

$$e^{At} = e^{2t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^{2t} \begin{bmatrix} 0 & -2 & -2 \\ 2 & 0 & -2 \\ -1 & -1 & 0 \end{bmatrix} + \frac{t^2 e^{2t}}{2} \begin{bmatrix} -2 & 2 & 4 \\ 2 & -2 & -4 \\ -2 & 2 & 4 \end{bmatrix}$$

$$y = e^{2t} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + te^{2t} \begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix} + \frac{t^2 e^{2t}}{2} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

[3] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & -1 & -1 \\ -1 & -2 & -2 \\ 2 & 1 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda = 0, 0, 0$$

$$e^{At} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + t \begin{bmatrix} 1 & -1 & -1 \\ -1 & -2 & -2 \\ 2 & 1 & 1 \end{bmatrix} + \frac{t^2}{2} \begin{bmatrix} 0 & 0 & 0 \\ -3 & 3 & 3 \\ 3 & -3 & -3 \end{bmatrix}$$

$$y = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ -4 \\ 5 \end{bmatrix} + \frac{t^2}{2} \begin{bmatrix} 0 \\ -3 \\ 3 \end{bmatrix}$$

[4] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & -1 & 1 \\ -1 & 2 & 2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda = 1, 1, 1$$

$$e^{At} = e^t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^t \begin{bmatrix} 1 & -2 & 3 \\ 1 & -2 & 1 \\ -1 & 2 & 1 \end{bmatrix} + \frac{t^2e^t}{2} \begin{bmatrix} -4 & 8 & 4 \\ -2 & 4 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$y = e^t \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + te^t \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} + \frac{t^2e^t}{2} \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$$

[5] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} -2 & -1 & 1 \\ 2 & 1 & -1 \\ 2 & 3 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$\lambda = 0, 0, 0$$

$$e^{At} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + t \begin{bmatrix} -2 & -1 & 1 \\ 2 & 1 & -1 \\ 2 & 3 & 1 \end{bmatrix} + \frac{t^2}{2} \begin{bmatrix} 4 & 4 & 0 \\ -4 & -4 & 0 \\ 4 & 4 & 0 \end{bmatrix}$$

$$y = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} + \frac{t^2}{2} \begin{bmatrix} 4 \\ -4 \\ 4 \end{bmatrix}$$

[6] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & 3 & 1 \\ -1 & 1 & 1 \\ 2 & -2 & -2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$\lambda = 0, 0, 0$$

$$e^{At} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + t \begin{bmatrix} 1 & 3 & 1 \\ -1 & 1 & 1 \\ 2 & -2 & -2 \end{bmatrix} + \frac{t^2}{2} \begin{bmatrix} 0 & 4 & 2 \\ 0 & -4 & -2 \\ 0 & 8 & 4 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + t \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} + \frac{t^2}{2} \begin{bmatrix} 4 \\ -4 \\ 8 \end{bmatrix}$$

[7] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} -2 & 3 & 2 \\ -1 & 2 & -2 \\ -1 & 1 & 3 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda = 1, 1, 1$$

$$e^{At} = e^t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^t \begin{bmatrix} -3 & 3 & 2 \\ -1 & 1 & -2 \\ -1 & 1 & 2 \end{bmatrix} + \frac{t^2 e^t}{2} \begin{bmatrix} 4 & -4 & -8 \\ 4 & -4 & -8 \\ 0 & 0 & 0 \end{bmatrix}$$

$$y = e^t \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + te^t \begin{bmatrix} -4 \\ -4 \\ 0 \end{bmatrix} + \frac{t^2 e^t}{2} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

[8] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & -1 & 2 \\ -1 & -1 & 2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda = 1, 1, 1$$

$$e^{At} = e^t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^t \begin{bmatrix} 1 & 1 & -1 \\ -2 & -2 & 2 \\ -1 & -1 & 1 \end{bmatrix} + \frac{t^2 e^t}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$y = e^t \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + te^t \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \frac{t^2 e^t}{2} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

3×3 Exercise Set J (symmetric matrices)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\lambda = 0, 2, 3 \qquad A^n = \frac{0^n}{6} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix} + \frac{2^n}{2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} + \frac{3^n}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$A = \begin{bmatrix} -3 & 1 & -1 \\ 1 & -2 & 0 \\ -1 & 0 & -2 \end{bmatrix}$$

$$\lambda = -4, -2, -1 \qquad A^n = \frac{(-4)^n}{6} \begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} + \frac{(-2)^n}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} + \frac{(-1)^n}{3} \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

[3] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\lambda = 0, 1, 3 \qquad A^n = \frac{0^n}{3} \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} + \frac{3^n}{6} \begin{bmatrix} 4 & 2 & -2 \\ 2 & 1 & -1 \\ -2 & -1 & 1 \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\lambda = 1, 3, 4 \qquad A^n = \frac{1}{6} \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ -1 & -2 & 1 \end{bmatrix} + \frac{3^n}{2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} + \frac{4^n}{3} \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

[5] Find A^n where A is the matrix

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -3 & -1 \\ 0 & -1 & -2 \end{bmatrix}$$

$$\lambda = -4, -2, -1 \qquad A^n \;=\; \frac{(-4)^n}{6}\begin{bmatrix} 1 & -2 & -1 \\ -2 & 4 & 2 \\ -1 & 2 & 1 \end{bmatrix} + \frac{(-2)^n}{2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} + \frac{(-1)^n}{3}\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

[6] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & -2 \end{bmatrix}$$

$$\lambda = -3, -1, 0 \qquad A^n \;=\; \frac{(-3)^n}{6}\begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & -2 \\ 2 & -2 & 4 \end{bmatrix} + \frac{(-1)^n}{2}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{0^n}{3}\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

[7] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} 3 & 0 & -1 \\ 0 & 3 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\lambda = 1, 3, 4 \qquad A^n \;=\; \frac{1}{6}\begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & -2 \\ 2 & -2 & 4 \end{bmatrix} + \frac{3^n}{2}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{4^n}{3}\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

[8] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

$$\lambda = 1, 2, 4 \qquad A^n \;=\; \frac{1}{3}\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix} + \frac{2^n}{2}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{4^n}{6}\begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & -2 \\ 2 & -2 & 4 \end{bmatrix}$$

3×3 Exercise Set K (symmetric matrices)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -3 & 1 & 0 \\ 1 & -2 & -1 \\ 0 & -1 & -3 \end{bmatrix}$$

$$\lambda = -4, -3, -1 \qquad e^{At} = \frac{e^{-4t}}{3}\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix} + \frac{e^{-3t}}{2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} + \frac{e^{-t}}{6}\begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ -1 & -2 & 1 \end{bmatrix}$$

[2] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -1 & -1 & -1 \\ -1 & -2 & 0 \\ -1 & 0 & -2 \end{bmatrix}$$

$$\lambda = -3, -2, 0 \qquad e^{At} = \frac{e^{-3t}}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{e^{-2t}}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

[3] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -2 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

$$\lambda = -3, -1, 0 \qquad e^{At} = \frac{e^{-3t}}{6}\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} + \frac{e^{-t}}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\lambda = 1, 2, 4 \qquad e^{At} = \frac{e^{t}}{3}\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} + \frac{e^{2t}}{2}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{e^{4t}}{6}\begin{bmatrix} 1 & -1 & -2 \\ -1 & 1 & 2 \\ -2 & 2 & 4 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\lambda = 0, 2, 3 \qquad e^{At} = \frac{1}{6}\begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & -2 \\ 2 & -2 & 4 \end{bmatrix} + \frac{e^{2t}}{2}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{e^{3t}}{3}\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -2 & 1 & -1 \\ 1 & -3 & 0 \\ -1 & 0 & -3 \end{bmatrix}$$

$$\lambda = -4, -3, -1 \qquad e^{At} = \frac{e^{-4t}}{3}\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} + \frac{e^{-3t}}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} + \frac{e^{-t}}{6}\begin{bmatrix} 4 & 2 & -2 \\ 2 & 1 & -1 \\ -2 & -1 & 1 \end{bmatrix}$$

[7] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\lambda = 0, 2, 3 \qquad e^{At} = \frac{1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix} + \frac{e^{2t}}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{e^{3t}}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

[8] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} -2 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 1 & -3 \end{bmatrix}$$

$$\lambda = -4, -2, -1 \qquad e^{At} = \frac{e^{-4t}}{6}\begin{bmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ -2 & -2 & 4 \end{bmatrix} + \frac{e^{-2t}}{2}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{e^{-t}}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

3×3 Exercise Set L (quadratic forms)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Express the quadratic form

$$2x^2 + 2y^2 + 2xz + 2yz + z^2$$

as a sum of squares of othogonal linear forms.

$$\lambda = 0, 2, 3 \qquad A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$(x - y)^2 + (x + y + z)^2$$

[2] Express the quadratic form

$$x^2 - 2xy + 2y^2 + 2yz + z^2$$

as a sum of squares of othogonal linear forms.

$$\lambda = 0, 1, 3 \qquad A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 & -2 & -1 \\ -2 & 4 & 2 \\ -1 & 2 & 1 \end{bmatrix}$$

$$\frac{1}{2}(x + z)^2 + \frac{1}{2}(x - 2y - z)^2$$

[3] Express the quadratic form

$$-3x^2 + 2xy - 2y^2 - 2yz - 3z^2$$

as a sum of squares of othogonal linear forms.

$$\lambda = -4, -3, -1 \qquad A = \begin{bmatrix} -3 & 1 & 0 \\ 1 & -2 & -1 \\ 0 & -1 & -3 \end{bmatrix} = -\frac{4}{3}\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix} - \frac{3}{2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} - \frac{1}{6}\begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ -1 & -2 & 1 \end{bmatrix}$$

$$-\frac{4}{3}(x - y - z)^2 - \frac{3}{2}(x + z)^2 - \frac{1}{6}(x + 2y - z)^2$$

[4] Express the quadratic form

$$-2x^2 - 2y^2 - 2xz + 2yz - 3z^2$$

as a sum of squares of othogonal linear forms.

$$\lambda = -4, -2, -1 \qquad A = \begin{bmatrix} -2 & 0 & -1 \\ 0 & -2 & 1 \\ -1 & 1 & -3 \end{bmatrix} = -\frac{2}{3}\begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & -2 \\ 2 & -2 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$-\frac{2}{3}(x - y + 2z)^2 - (x + y)^2 - \frac{1}{3}(x - y - z)^2$$

[5] Express the quadratic form

$$-x^2 - y^2 + 2xz - 2yz - 2z^2$$

as a sum of squares of othogonal linear forms.

$$\lambda = -3, -1, 0 \qquad A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & -1 & -2 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 1 & -1 & -2 \\ -1 & 1 & 2 \\ -2 & 2 & 4 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$-\frac{1}{2}(x - y - 2z)^2 - \frac{1}{2}(x + y)^2$$

[6] Express the quadratic form

$$2x^2 - 2xy + y^2 + 2xz + z^2$$

as a sum of squares of othogonal linear forms.

$$\lambda = 0, 1, 3 \qquad A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

$$\frac{1}{2}(y + z)^2 + \frac{1}{2}(2x - y + z)^2$$

[7] Express the quadratic form

$$2x^2 - 2xy + y^2 + 2yz + 2z^2$$

as a sum of squares of othogonal linear forms.

$$\lambda = 0, 2, 3 \qquad A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$(x + z)^2 + (x - y - z)^2$$

[8] Express the quadratic form

$$-x^2 + 2xy - 2y^2 - 2yz - z^2$$

as a sum of squares of othogonal linear forms.

$$\lambda = -3, -1, 0 \qquad A = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & -1 \\ 0 & -1 & -1 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 1 & -2 & -1 \\ -2 & 4 & 2 \\ -1 & 2 & 1 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$-\frac{1}{2}(x - 2y - z)^2 - \frac{1}{2}(x + z)^2$$