

## 2 × 2 Exercise Set A (distinct roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix}$$

[2] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & -1 \\ -3 & -1 \end{bmatrix}$$

[3] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

[4] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

[5] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 2 \\ -2 & -3 \end{bmatrix}$$

[6] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}$$

[7] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

[8] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

**2 × 2 Exercise Set B (distinct roots)**

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 3 & 1 \\ -3 & -1 \end{bmatrix}$$

[2] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -3 & 2 \\ -3 & 2 \end{bmatrix}$$

[3] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -1 & -3 \\ -2 & 0 \end{bmatrix}$$

[4] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 3 & 3 \\ -1 & -1 \end{bmatrix}$$

[5] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & -1 \\ -3 & -1 \end{bmatrix}$$

[6] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -2 & 2 \\ -3 & 3 \end{bmatrix}$$

[7] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & -2 \\ 3 & -3 \end{bmatrix}$$

[8] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 0 & -1 \\ -3 & 2 \end{bmatrix}$$

**2 × 2 Exercise Set C (distinct roots)**

Linear Algebra, Dave Bayer, November 24, 2013

[1] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

[2] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 0 & -1 \\ 2 & -3 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

[3] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 1 & -1 \\ -3 & -1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

[4] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

[5] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

[6] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} -3 & 2 \\ -3 & 2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

[7] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} -2 & -1 \\ -2 & -1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

[8] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} -1 & -2 \\ 1 & 2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

## 2 × 2 Exercise Set D (repeated roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 3 \\ -3 & -5 \end{bmatrix}$$

[2] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

[3] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$$

[4] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & -1 \\ 4 & 6 \end{bmatrix}$$

[5] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 6 & 5 \\ -5 & -4 \end{bmatrix}$$

[6] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 6 & 1 \\ -1 & 4 \end{bmatrix}$$

[7] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix}$$

[8] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} -5 & -4 \\ 1 & -1 \end{bmatrix}$$

## 2 × 2 Exercise Set E (repeated roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 5 & -4 \\ 4 & -3 \end{bmatrix}$$

[2] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -5 & -4 \\ 4 & 3 \end{bmatrix}$$

[3] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -3 & -1 \\ 1 & -1 \end{bmatrix}$$

[4] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 5 & 1 \\ -1 & 3 \end{bmatrix}$$

[5] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

[6] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -4 & 3 \\ -3 & 2 \end{bmatrix}$$

[7] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}$$

[8] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -1 & 2 \\ -2 & -5 \end{bmatrix}$$

## $2 \times 2$ Exercise Set F (repeated roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 5 & 4 \\ -4 & -3 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

[2] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 2 & -4 \\ 1 & 6 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

[3] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} -1 & 4 \\ -1 & -5 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

[4] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

[5] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} -2 & -1 \\ 4 & 2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

[6] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

[7] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} -5 & -4 \\ 4 & 3 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

[8] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} -4 & -3 \\ 3 & 2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

## 2 × 2 Exercise Set G (symmetric matrices)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} -3 & -2 \\ -2 & -3 \end{bmatrix}$$

[2] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} -1 & 2 \\ 2 & 2 \end{bmatrix}$$

[3] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$$

[4] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} -4 & -2 \\ -2 & -1 \end{bmatrix}$$

[5] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}$$

[6] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix}$$

[7] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} -4 & 1 \\ 1 & -4 \end{bmatrix}$$

[8] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}$$

## 2 × 2 Exercise Set H (symmetric matrices)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -3 & -2 \\ -2 & 0 \end{bmatrix}$$

[2] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$$

[3] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$$

[4] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix}$$

[5] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

[6] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -1 & -2 \\ -2 & 2 \end{bmatrix}$$

[7] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -5 & -2 \\ -2 & -2 \end{bmatrix}$$

[8] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -2 & -2 \\ -2 & -5 \end{bmatrix}$$



## 2 × 2 Exercise Set I (quadratic forms)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Express the quadratic form

$$3x^2 - 2xy + 3y^2$$

as a sum of squares of orthogonal linear forms.

[2] Express the quadratic form

$$-3x^2 + 2xy - 3y^2$$

as a sum of squares of orthogonal linear forms.

[3] Express the quadratic form

$$-x^2 - 4xy - y^2$$

as a sum of squares of orthogonal linear forms.

[4] Express the quadratic form

$$2x^2 - 4xy + 5y^2$$

as a sum of squares of orthogonal linear forms.

[5] Express the quadratic form

$$2x^2 + 4xy - y^2$$

as a sum of squares of orthogonal linear forms.

[6] Express the quadratic form

$$3x^2 + 2xy + 3y^2$$

as a sum of squares of orthogonal linear forms.

[7] Express the quadratic form

$$-2x^2 + 4xy + y^2$$

as a sum of squares of orthogonal linear forms.

[8] Express the quadratic form

$$-x^2 - 8xy - y^2$$

as a sum of squares of orthogonal linear forms.

## 2 × 2 Exercise Set J (recurrence relations)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = -5f(n-1) - 4f(n-2)$$

[2] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = 6f(n-1) - 5f(n-2)$$

[3] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = -6f(n-1) - 8f(n-2)$$

[4] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = -4f(n-1) + 5f(n-2)$$

[5] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = -f(n-1) + 6f(n-2)$$

[6] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = -6f(n-1) + 7f(n-2)$$

[7] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = -6f(n-1) - 5f(n-2)$$

[8] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = f(n-1) + 6f(n-2)$$

### $3 \times 3$ Exercise Set A (distinct roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

[2] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

[3] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

[4] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

[5] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

[6] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

[7] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 2 & 2 & 2 \end{bmatrix}$$

[8] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 2 \\ 2 & 1 & 1 \end{bmatrix}$$

**3 × 3 Exercise Set B (distinct roots)**

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

[2] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

[3] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

[4] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

[5] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

[6] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

[7] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

[8] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

**3 × 3 Exercise Set C (distinct roots)**

Linear Algebra, Dave Bayer, November 24, 2013

[1] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

[2] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 0 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

[3] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \\ 0 & 2 & 2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

[4] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 1 \\ 2 & 2 & 1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

[5] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

[6] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

[7] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

[8] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

### 3 × 3 Exercise Set D (repeated roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

[2] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

[3] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

[4] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

[5] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

[6] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

[7] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix}$$

[8] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

## 3 × 3 Exercise Set E (repeated roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

[2] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

[3] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

[4] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

[5] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

[6] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

[7] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

[8] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

**3 × 3 Exercise Set F (repeated roots)**

Linear Algebra, Dave Bayer, November 24, 2013

[1] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

[2] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

[3] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

[4] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

[5] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

[6] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

[7] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

[8] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$



**3 × 3 Exercise Set G (identical roots)**

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 3 & 3 \\ -2 & 3 & 1 \\ 2 & -1 & 1 \end{bmatrix}$$

[2] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 1 \\ 1 & -2 & -2 \end{bmatrix}$$

[3] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 3 & 3 & -2 \\ -1 & -1 & -2 \\ 1 & 1 & -2 \end{bmatrix}$$

[4] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -1 \\ 1 & 3 & 1 \end{bmatrix}$$

[5] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 3 & 2 & 2 \\ -2 & -1 & -2 \\ 3 & 3 & 1 \end{bmatrix}$$

[6] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & -1 & 1 \\ -1 & -2 & -1 \end{bmatrix}$$

[7] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} -1 & 2 & 1 \\ -1 & 2 & 2 \\ 1 & -1 & 2 \end{bmatrix}$$

[8] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & -2 & -1 \\ 2 & 2 & -1 \\ -2 & -2 & 2 \end{bmatrix}$$

### $3 \times 3$ Exercise Set H (identical roots)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & 1 & -1 \\ -1 & 1 & 3 \end{bmatrix}$$

[2] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -2 & 2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & 1 \end{bmatrix}$$

[3] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -1 & 3 & 2 \\ 2 & 1 & -2 \\ -2 & 3 & 3 \end{bmatrix}$$

[4] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -2 & 2 & 2 \\ 1 & -2 & 1 \\ -2 & 2 & -2 \end{bmatrix}$$

[5] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -2 & -1 & 1 \\ -1 & -2 & 3 \\ -1 & -1 & 1 \end{bmatrix}$$

[6] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 1 & 2 & -2 \\ 1 & 2 & 2 \end{bmatrix}$$

[7] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 3 & 2 & 2 \\ -2 & -1 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

[8] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & -2 & -2 \\ -1 & 2 & 1 \\ 2 & 2 & 2 \end{bmatrix}$$

**3 × 3 Exercise Set I (identical roots)**

Linear Algebra, Dave Bayer, November 24, 2013

[1] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} -1 & 3 & 1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

[2] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 2 & -2 & -2 \\ 2 & 2 & -2 \\ -1 & -1 & 2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

[3] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 1 & -1 & -1 \\ -1 & -2 & -2 \\ 2 & 1 & 1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

[4] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & -1 & 1 \\ -1 & 2 & 2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

[5] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} -2 & -1 & 1 \\ 2 & 1 & -1 \\ 2 & 3 & 1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

[6] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 1 & 3 & 1 \\ -1 & 1 & 1 \\ 2 & -2 & -2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

[7] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} -2 & 3 & 2 \\ -1 & 2 & -2 \\ -1 & 1 & 3 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

[8] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & -1 & 2 \\ -1 & -1 & 2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

## 3 × 3 Exercise Set J (symmetric matrices)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

[2] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} -3 & 1 & -1 \\ 1 & -2 & 0 \\ -1 & 0 & -2 \end{bmatrix}$$

[3] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

[4] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

[5] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -3 & -1 \\ 0 & -1 & -2 \end{bmatrix}$$

[6] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & -2 \end{bmatrix}$$

[7] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 3 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

[8] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

### 3 × 3 Exercise Set K (symmetric matrices)

Linear Algebra, Dave Bayer, November 24, 2013

[1] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -3 & 1 & 0 \\ 1 & -2 & -1 \\ 0 & -1 & -3 \end{bmatrix}$$

[2] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -1 & -1 & -1 \\ -1 & -2 & 0 \\ -1 & 0 & -2 \end{bmatrix}$$

[3] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -2 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

[4] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

[5] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

[6] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -2 & 1 & -1 \\ 1 & -3 & 0 \\ -1 & 0 & -3 \end{bmatrix}$$

[7] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

[8] Find  $e^{A^t}$  where  $A$  is the matrix

$$A = \begin{bmatrix} -2 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 1 & -3 \end{bmatrix}$$

**3 × 3 Exercise Set L (quadratic forms)**

Linear Algebra, Dave Bayer, November 24, 2013

[1] Express the quadratic form

$$2x^2 + 2y^2 + 2xz + 2yz + z^2$$

as a sum of squares of orthogonal linear forms.

[2] Express the quadratic form

$$x^2 - 2xy + 2y^2 + 2yz + z^2$$

as a sum of squares of orthogonal linear forms.

[3] Express the quadratic form

$$-3x^2 + 2xy - 2y^2 - 2yz - 3z^2$$

as a sum of squares of orthogonal linear forms.

[4] Express the quadratic form

$$-2x^2 - 2y^2 - 2xz + 2yz - 3z^2$$

as a sum of squares of orthogonal linear forms.

[5] Express the quadratic form

$$-x^2 - y^2 + 2xz - 2yz - 2z^2$$

as a sum of squares of orthogonal linear forms.

[6] Express the quadratic form

$$2x^2 - 2xy + y^2 + 2xz + z^2$$

as a sum of squares of orthogonal linear forms.

[7] Express the quadratic form

$$2x^2 - 2xy + y^2 + 2yz + 2z^2$$

as a sum of squares of orthogonal linear forms.

[8] Express the quadratic form

$$-x^2 + 2xy - 2y^2 - 2yz - z^2$$

as a sum of squares of orthogonal linear forms.