Final Exam

Linear Algebra, Dave Bayer, December 16-22, 2021

Name: Solutions **Uni:**

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	**Total**

If you need more that one page for a problem, clearly indicate on each page where to look next for your work.

[1] Find a system of equations having as solution set the following affine subspace of $\mathbb{R}^3$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} t$$

rank 1, need 2 equations

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \left(\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} t \right) = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\to 0$$

$$\boxed{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}}$$

[2] Find the orthogonal projection of the vector $(1, 1, 0, 0)$ onto the subspace of $\mathbb{R}^4$ spanned by the vectors $(1, 1, 1, 1)$ and $(0, 1, 1, 1)$.

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \end{bmatrix} 2/3 = \begin{bmatrix} 4 & 3 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -2/3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} / 3$$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} / 3 = \boxed{\begin{bmatrix} 3 \\ 1 \\ 1 \\ 1 \end{bmatrix} / 3}$$

Check:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 3 \\ 1 \\ 1 \\ 1 \end{bmatrix} / 3 \right) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 0 \quad ☑$$

[3] Consider $\mathbb{R}^3$ equipped with the inner product

$$\langle (a,b,c),(d,e,f)\rangle = \begin{bmatrix} a & b & c \end{bmatrix}\begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} d \\ e \\ f \end{bmatrix}$$

Find an orthogonal basis for $\mathbb{R}^3$ with respect to this inner product.

$\boxed{v_1 = [1\ 0\ 0]}$

$$[1\ 0\ 0]\begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = [3\ 0\ 1]\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$[3\ 0\ 1]\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 0$$

$\boxed{v_2 = [0\ 1\ 0]}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} = 0$$

$\boxed{v_3 = [1\ 0\ -3]}$

Check:

$$\begin{array}{|c|c|c|} \hline 3 & 0 & 1 \\ \hline 0 & 2 & 0 \\ \hline 1 & 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{c|ccc} \times & 0 & 1 & 0 \\ \hline 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}$$

0 ✓

$$\begin{array}{c|ccc} \times & 1 & 0 & -3 \\ \hline 1 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}$$

0 ✓

$$\begin{array}{c|ccc} \times & 1 & 0 & -3 \\ \hline 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{array}$$

0 ✓

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 1 & -3 \\ -1 & -1 \end{bmatrix}$$

$$\lambda = -2, 2 \qquad A^n = \frac{(-2)^n}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} + \frac{2^n}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}$$

$$r + s = 0$$
$$rs = 1(-1) - (-3)(-1) = -4 \quad \Rightarrow \quad r = -2,\ s = 2$$

$$\begin{array}{ccccc}
\lambda & w & A - \lambda I & v & \\
-2 & [1\ 3] & \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \end{bmatrix}[1\ 3]/4 = \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}/4 \\
2 & [1\ -1] & \begin{bmatrix} -1 & -3 \\ -1 & -3 \end{bmatrix} & \begin{bmatrix} 3 \\ -1 \end{bmatrix} & \begin{bmatrix} 3 \\ -1 \end{bmatrix}[1\ -1]/4 = \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}/4
\end{array}$$

$$\boxed{A^n = (-2)^n \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}/4 + 2^n \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}/4}$$

check:

$$I = A^0 = \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}/4 + \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}/4 \quad \checkmark$$

$$A = A^1 = (-2)\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}/4 + 2\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}/4$$

$$\begin{array}{c|c} -2 \quad 6 & -6 \quad -6 \\ \hline -2 \quad -2 & -6 \quad 2 \end{array} /4 = \begin{bmatrix} 1 & -3 \\ -1 & -1 \end{bmatrix} \quad \checkmark$$

[5] Solve the differential equation $y' = Ay$ where

$$A \; = \; \begin{bmatrix} 0 & -3 \\ -1 & 2 \end{bmatrix}, \qquad y(0) \; = \; \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\lambda = -1, 3 \qquad e^{At} \; = \; \frac{e^{-t}}{4} \begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix} + \frac{e^{3t}}{4} \begin{bmatrix} 1 & -3 \\ -1 & 3 \end{bmatrix} \qquad y \; = \; \frac{e^{-t}}{4} \begin{bmatrix} 9 \\ 3 \end{bmatrix} + \frac{e^{3t}}{4} \begin{bmatrix} -5 \\ 5 \end{bmatrix}$$

$$\begin{aligned} r+s &= 2 \\ rs &= -3 \end{aligned} \implies r=-1,\ s=3$$

λ	$A-\lambda I$	v
-1	$\begin{bmatrix} 1 & -3 \\ -1 & 3 \end{bmatrix}$	$\begin{bmatrix} 3 \\ 1 \end{bmatrix}$
3	$\begin{bmatrix} -3 & -3 \\ -1 & -1 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\times 3/4$ (can't help with sum)

$1+2=3 \qquad 3+1=4 \qquad -1+1=0$

$$y(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{3}{4}\begin{bmatrix} 3 \\ 1 \end{bmatrix} + \frac{5}{4}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

or solve

$$\begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}\Big/4 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}\Big/4$$

$\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ sees $y' = -y$, $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ sees $y' = 3y$ so

$$\boxed{y(t) = \frac{3}{4}e^{-t}\begin{bmatrix} 3 \\ 1 \end{bmatrix} + \frac{5}{4}e^{3t}\begin{bmatrix} -1 \\ 1 \end{bmatrix}}$$

check:

$$y'(t) = -\frac{3}{4}e^{-t}\begin{bmatrix} 3 \\ 1 \end{bmatrix} + 3\frac{5}{4}e^{3t}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$y'(0) = -\frac{3}{4}\begin{bmatrix} 3 \\ 1 \end{bmatrix} + 3\frac{5}{4}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -24 \\ 12 \end{bmatrix}\Big/4 = \begin{bmatrix} -6 \\ 3 \end{bmatrix}$$

$$Ay(0) = \begin{bmatrix} 0 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -6 \\ 3 \end{bmatrix} \checkmark$$

[6] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = 7\,f(n-1) \;-\; 6\,f(n-2)$$

$$\begin{bmatrix} f(n+1) \\ f(n) \end{bmatrix} = \begin{bmatrix} 7 & -6 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} b \\ a \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -1 & 6 \\ -1 & 6 \end{bmatrix}\begin{bmatrix} b \\ a \end{bmatrix} + \frac{6^n}{5}\begin{bmatrix} 6 & -6 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} b \\ a \end{bmatrix}$$

$$f(n) \;=\; \frac{1}{5}(\,-\,b \,+\, 6a) \,+\, \frac{6^n}{5}(b \,-\, a)$$

$$\begin{bmatrix} f(2) \\ f(1) \end{bmatrix} = \begin{bmatrix} 7 & -6 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} f(1) \\ f(0) \end{bmatrix}$$

$$\begin{bmatrix} 7 & -6 \\ 1 & 0 \end{bmatrix} \quad \begin{matrix} r+s=7 \\ rs=6 \end{matrix} \quad \Rightarrow \; r=1,\; s=6$$

$$\begin{bmatrix} f(n+1) \\ f(n) \end{bmatrix} = \begin{bmatrix} 7 & -6 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} f(1) \\ f(0) \end{bmatrix}$$

λ	w	$A-\lambda I$	v	
1	$[-1\ 6]$	$\begin{bmatrix} 6 & -6 \\ 1 & -1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}[-1\ 6]/5 = \begin{bmatrix} -1 & 6 \\ -1 & 6 \end{bmatrix}/5$
6	$[1\ -1]$	$\begin{bmatrix} 1 & -6 \\ 1 & -6 \end{bmatrix}$	$\begin{bmatrix} 6 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 6 \\ 1 \end{bmatrix}[1\ -1]/5 = \begin{bmatrix} 6 & -6 \\ 1 & -1 \end{bmatrix}/5$

$$\begin{bmatrix} 7 & -6 \\ 1 & 0 \end{bmatrix}^n = 1^n\begin{bmatrix} -1 & 6 \\ -1 & 6 \end{bmatrix}/5 + 6^n\begin{bmatrix} 6 & -6 \\ 1 & -1 \end{bmatrix}/5$$

$$\begin{bmatrix} f(n+1) \\ f(n) \end{bmatrix} = \begin{bmatrix} 7 & -6 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} b \\ a \end{bmatrix} = 1^n\begin{bmatrix} -1 & 6 \\ -1 & 6 \end{bmatrix}/5\begin{bmatrix} b \\ a \end{bmatrix} + 6^n\begin{bmatrix} 6 & -6 \\ 1 & -1 \end{bmatrix}/5\begin{bmatrix} b \\ a \end{bmatrix}$$

$$\boxed{f(n) = \frac{1}{5}(-b+6a) + \frac{6^n}{5}(b-a)}$$

check example:

n	$f(n)$
0	1
1	2
2	$8 = 7\cdot 2 - 6\cdot 1$

$$f(2) = \frac{1}{5}(-2+6\cdot 1) + \frac{36}{5}(2-1)$$

$$= \frac{40}{5} = 8 \quad ✓$$

[7] Express the quadratic form

$$x^2 + 2xy + 2y^2 - 2yz + z^2$$

as a sum of squares of orthogonal linear forms.

$$\lambda = 0, 1, 3 \qquad A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ -1 & -2 & 1 \end{bmatrix}$$

$$\frac{1}{2}(x+z)^2 + \frac{1}{2}(x+2y-z)^2$$

$$[x\ y\ z]\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = x^2+2xy+2y^2-2yz+z^2$$

λ w $A-\lambda I$ v

$0 \quad [-1\ 1\ 1]\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ (matrix looked singular)

$1 \quad [1\ 0\ 1]\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ (subtracting 1 looked promising)

$3 \quad [1\ 2\ -1]\begin{bmatrix} -2 & 1 & 0 \\ 1 & -1 & -1 \\ 0 & -1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ ($0+1+3 =$ trace)

$$[x\ y\ z]\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}[x\ y\ z]\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}[1\ 0\ 1]\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
$$+ \frac{3}{6}[x\ y\ z]\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}[1\ 2\ -1]\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\boxed{\frac{1}{2}(x+z)^2 + \frac{1}{2}(x+2y-z)^2}$$

check:

x^2	y^2	z^2	xy	xz	yz	
1		1		2		$(x+z)^2$
1	4	1	4	−2	−4	$(x+2y-z)^2$
1	2	1	2	0	−2	average $\frac{1}{2}+\frac{1}{2}$

✓

[8] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\lambda = 4, 1, 1 \qquad e^{At} = \frac{e^{4t}}{9}\begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} + \frac{e^{t}}{9}\begin{bmatrix} 5 & -4 & -4 \\ -2 & 7 & -2 \\ -3 & -3 & 6 \end{bmatrix} + \frac{te^{t}}{3}\begin{bmatrix} -1 & -1 & 2 \\ 1 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$r+s+t = 6$

$rs+rt+st = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} = 3+2+4 = 9 \qquad 1, 1, 4$

$rst = 2\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 2(-1) + 2\cdot 3 = 4$

$$\begin{bmatrix} 1 & 1 & \\ & 1 & \\ & & 4 \end{bmatrix} = 1\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & \\ & 0 & \\ & & 0 \end{bmatrix} + 4\begin{bmatrix} 0 & & \\ & 0 & \\ & & 1 \end{bmatrix}$$ (model in good coords)

① $\lambda = 4 \quad [1\ 1\ 1]\begin{bmatrix} -2 & 1 & 2 \\ 1 & -2 & 0 \\ 1 & 1 & -2 \end{bmatrix} \quad \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix} \qquad \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}[1\ 1\ 1]/9$

$$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix} = 1\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} + \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} + 4\begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}/9$$ (work out in steps in our coords)

② $\begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}/9 + \begin{bmatrix} 5 & -4 & -4 \\ -2 & 7 & -2 \\ -3 & -3 & 6 \end{bmatrix}/9 = I$

$$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix} = 1\begin{bmatrix} 5 & -4 & -4 \\ -2 & 7 & -2 \\ -3 & -3 & 6 \end{bmatrix}/9 + \begin{bmatrix} -3 & -3 & 6 \\ 3 & 3 & -6 \\ 0 & 0 & 0 \end{bmatrix}/9 + 4\begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}/9$$

③ $\begin{array}{c|c|c} 18-5-16 & 9+4-16 & 18+4-16 \\ \hline 9+2-8 & 18-7-8 & 2-8 \\ \hline 9+3-12 & 9+3-12 & 18-6-12 \end{array} /9$

$$\boxed{e^{At} = e^{t}\begin{bmatrix} 5 & -4 & -4 \\ -2 & 7 & -2 \\ -3 & -3 & 6 \end{bmatrix}/9 + te^{t}\begin{bmatrix} -3 & -3 & 6 \\ 3 & 3 & -6 \\ 0 & 0 & 0 \end{bmatrix}/9 + e^{4t}\begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}/9}$$

check: $\begin{bmatrix} 0 & 1 & \\ & 0 & \\ & & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -3 & -3 & 6 \\ 3 & 3 & -6 \\ 0 & 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ ✓