Final Exam

Linear Algebra, Dave Bayer, December 16-22, 2021

[1] Find a system of equations having as solution set the following affine subspace of $\mathbb{R}^3$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} t$$

[2] Find the orthogonal projection of the vector $(1, 1, 0, 0)$ onto the subspace of $\mathbb{R}^4$ spanned by the vectors $(1, 1, 1, 1)$ and $(0, 1, 1, 1)$.

[3] Consider $\mathbb{R}^3$ equipped with the inner product

$$\langle (a, b, c), (d, e, f) \rangle = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} d \\ e \\ f \end{bmatrix}$$

Find an orthogonal basis for $\mathbb{R}^3$ with respect to this inner product.

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 1 & -3 \\ -1 & -1 \end{bmatrix}$$

[5] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 0 & -3 \\ -1 & 2 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

[6] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = 7\,f(n-1) - 6\,f(n-2)$$

[7] Express the quadratic form

$$x^2 + 2xy + 2y^2 - 2yz + z^2$$

as a sum of squares of orthogonal linear forms.

[8] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$