Exam 2, 8:40am

Linear Algebra, Dave Bayer, November 4, 2021

[1] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 2 & 0 \end{bmatrix}$$

[2] Find the orthogonal projection of the vector $(1, 0, 0, 0)$ onto the subspace of $\mathbb{R}^4$ spanned by the vectors $(1, 1, 1, 1)$ and $(0, 1, 2, 1)$.

[3] Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 3 & -1 & 0 & 0 \\ 0 & 1 & 4 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

[4] Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 1 & 5 \\ 0 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

[5] Consider $\mathbb{R}^3$ equipped with the inner product

$$\langle (a, b, c), (d, e, f) \rangle \;=\; \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} d \\ e \\ f \end{bmatrix}$$

Find an orthogonal basis for $\mathbb{R}^3$ with respect to this inner product.