Final Exam

Linear Algebra, Dave Bayer, December 17-23, 2020

[1] Find an orthogonal basis for $\mathbb{R}^4$ that includes the vector $(1, 0, 0, 1)$.

[2] Find a system of equations having as solution set the image of the following map from $\mathbb{R}^3$ to $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 2 & 2 \\ 1 & 0 & -1 \\ -1 & -2 & -1 \\ 0 & -2 & -2 \end{bmatrix} \begin{bmatrix} r \\ s \\ t \end{bmatrix}$$

[3] Let V be the subspace of $\mathbb{R}^4$ defined by the system of equations

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 2 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Find the 4×4 matrix A that projects $\mathbb{R}^4$ orthogonally onto V.

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 2 & 2 & 2 \end{bmatrix}$$

[6] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 4 & -1 \\ 4 & 0 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

[7] Express the quadratic form

$$3x^2 + 3y^2 + 2xz - 2yz + 2z^2$$

as a sum of squares of orthogonal linear forms.

[8] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[\,] \qquad \begin{bmatrix} 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ -4 & 0 & 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ -4 & 0 & 3 & 1 \\ 0 & -4 & 0 & 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 \\ -4 & 0 & 3 & 1 & 0 \\ 0 & -4 & 0 & 3 & 1 \\ 0 & 0 & -4 & 0 & 3 \end{bmatrix}$$

For example, $f(4) = 57$ and $f(5) = 135$.
Find a recurrence relation for $f(n)$.
Using Jordan canonical form, solve this recurrence relation to find a closed form formula for $f(n)$.