Exam 2

Linear Algebra, Dave Bayer, November 6-9, 2020

[1] Find an orthogonal basis for $\mathbb{R}^4$ that includes the vector $(1, 1, 1, 0)$.

[2] Let V be the vector space of all polynomials of degree $\leqslant 2$ in the variable x with coefficients in $\mathbb{R}$. Find an orthogonal basis for V with respect to the inner product

$$\langle f, g \rangle \;=\; \int_0^1 f(x)g(x)\,dx$$

[3] By least squares, find the equation of the form $z = ax + by + c$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \\ x_4 & y_4 & z_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

[4] Let V be the subspace of $\mathbb{R}^4$ defined by the system of equations

$$\begin{bmatrix} -1 & 2 & -1 & 0 \\ -1 & 1 & 1 & -1 \\ 0 & -1 & 2 & -1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Find the 4×4 matrix A that projects $\mathbb{R}^4$ orthogonally onto V.

[5] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$\begin{bmatrix} 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Find $f(3)$, $f(4)$, and $f(5)$. Find a recurrence relation for $f(n)$.