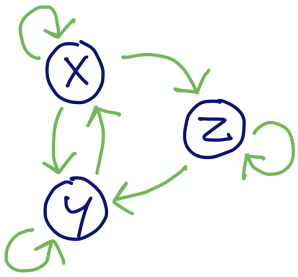
Exam 1

Linear Algebra, Dave Bayer, October 2-5, 2020

[1] Using matrix multiplication, count the number of paths of length six from x to y.



[2] Find all solutions to the system of equations

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ -1 & 3 & 2 & 1 \\ 0 & 4 & 4 & 4 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$$

[3] Find a system of equations having as solution set the image of the following map from $\mathbb{R}^3$ to $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 & -1 & 0 \\ 2 & 1 & -1 \\ -1 & 1 & 2 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} r \\ s \\ t \end{bmatrix}$$

[4] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 3 \\ 0 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

[5] Express $\begin{bmatrix} s \\ t \end{bmatrix}$ in terms of $\begin{bmatrix} a \\ b \end{bmatrix}$ for the following parametrizations of the same affine subspace of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 3 \\ 0 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} \qquad \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 6 & 9 \\ 4 & 6 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$