

Name _____ Uni _____

[1] Find a basis for the subspace V of \mathbb{R}^4 spanned by the vectors

$(1, 1, 0, 1)$ $(1, 0, 1, 1)$ $(2, 1, 1, 2)$ $(3, 2, 1, 3)$ $(3, 1, 2, 3)$ $(4, 2, 2, 4)$

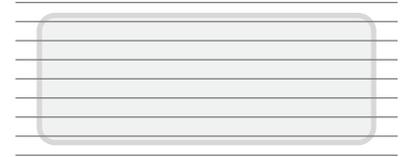
Extend this basis to a basis for \mathbb{R}^4 .

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Test 1

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[1] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}$$

$y = \boxed{} x + \boxed{}$

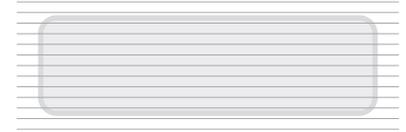
[4] Find the 4×4 matrix that projects orthogonally onto the plane spanned by the vectors $(1, 0, 1, 0)$ and $(0, 1, 0, 1)$.

$$A = \frac{1}{\boxed{}} \begin{bmatrix} \boxed{} & \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} & \boxed{} \end{bmatrix}$$



Test 1

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[1] Find the determinant of the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 & 3 \\ 0 & 1 & 1 & 5 & 5 \\ 2 & 2 & 2 & 2 & 2 \\ 0 & 1 & 1 & 1 & 7 \end{bmatrix}$$

$\det(A) =$



Test 1

[2] Find the inverse of the matrix

$$A = \begin{bmatrix} 5 & 2 & 1 \\ 3 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\square} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

(Do not write a negative denominator.)



Test 1

[3] Using Cramer's rule, solve for y in the system of equations

$$\begin{bmatrix} a & 1 & 2 \\ b & 1 & 1 \\ c & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$$

$$y = \frac{(\quad) a + (\quad) b + (\quad) c}{(\quad) a + (\quad) b + (\quad) c}$$



Test 1

[4] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[2] \quad \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 2 & 5 & 0 \\ 1 & 2 & 5 \\ 0 & 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 2 & 5 & 0 & 0 \\ 1 & 2 & 5 & 0 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Find $f(1)$ and $f(2)$. Find a recurrence relation for $f(n)$. Find $f(6)$.

$f(1) = $	<input type="text"/>	$f(2) = $	<input type="text"/>	
$f(n) = $	<input type="text"/>	$f(n-1) + $	<input type="text"/>	$f(n-2)$
$f(6) = $	<input type="text"/>			

[2] Find the 3×3 matrix A such that

$$A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad A \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \quad A \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

$$A = \frac{1}{\square} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$



Test 1

[3] Consider \mathbb{R}^3 equipped with the inner product

$$\langle (a, b, c), (d, e, f) \rangle = [a \ b \ c] \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} d \\ e \\ f \end{bmatrix}$$

Using this inner product, find the orthogonal projection of the vector $(3, 3, 3)$ onto the plane spanned by $(1, 0, 0)$ and $(0, 1, 0)$.

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Test 1

[3] Consider \mathbb{R}^3 equipped with the inner product

$$\langle (a, b, c), (d, e, f) \rangle = [a \ b \ c] \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} d \\ e \\ f \end{bmatrix}$$

Using this inner product, find the orthogonal projection of the vector $(2, 2, 2)$ onto the plane spanned by $(1, 0, 1)$ and $(0, 1, 1)$.

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