Our second exam will be held in class on Thursday, November 9, 2017.

Exam 2 will consist of five questions. The following are the skills that one needs to learn for this exam:

- Use least squares to fit data.
- Find the orthogonal projection of a vector to a subspace of $\mathbb{R}^n$.
- Find an orthogonal basis for a subspace of $\mathbb{R}^n$. Extend this independent set to an orthogonal basis for $\mathbb{R}^n$.
- Compute the determinant of an $n \times n$ matrix, using any or a combination of the methods we have learned in class: Directly using an understanding of the formula as permutations, expansion by minors, tracking the effect of Gaussian elimination.
- Compute the inverse of an $n \times n$ matrix, using the formula. Note that we have studied streamlined ways to carry out this computation for 2×2 and 3×3 matrices.
- Compute a 3×3 matrix A from a description of the linear map, using change of coordinates and the formula for the 3×3 inverse.
- Using Cramer's rule, solve for one value or a ratio of values in a system of equations.
- Express a recurrence relation as a matrix, and solve for a specific value by taking a matrix power.
- Find a recurrence relation describing a sequence of determinants, and solve for a specific value.
- Find the characteristic equation and a system of eigenvalues and eigenvectors for a 2×2 matrix. (The eigenvalues will be distinct integers.)

This material is covered in chapters five, six, and seven of Bretscher, and in past exam problems. You are encouraged to read the chapters in Bretscher carefully.

Homework will count as 10% of your course grade. The homework for Exam 2 is given below. You may turn in problems in batchs as you complete them; homework that is received early will receive more careful feedback. All homework must be submitted on or before the day of our exam. There is a homework box on the fourth floor of the Mathematics building for your section of Linear Algebra. Please turn in homework to the box corresponding to your section. Please write your uni very clearly on each page of homework.

Please hand in the following problems; they are the same in both the 5e and 4e editions of Bretscher. (You are encouraged to work similar problems for your own practice.)

- 5.1 [16], 5.2 [14], 5.3 [36], 5.4 [22], 5.5 [10]
- 6.1 [30], 6.2 [10], 6.3 [14]
- 7.1 [10], 7.2 [14], 7.3 [22], 7.4 [42], 7.5 [14]

What follows on the remaining pages of this study guide are practice problems for our second exam, taken from past semesters of the course. Note that some 3×3 matrix problems also appeared on our previous study guide. These problems should now be easier.

The sources for the following problems, along with many solutions, can be found on our Linear Algebra Course Materials page:

https://www.math.columbia.edu/~bayer/LinearAlgebra/

(F16 8:40 Exam 1) (Solutions)

[4] By least squares, find the equation of the form $z = ax + by$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(Note that b is multiplied by y in this equation.)

[5] Let V be the vector space $\mathbb{R}^3$, equipped with the inner product

$$\langle (a,b,c),(d,e,f) \rangle = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} d \\ e \\ f \end{bmatrix}$$

Using this inner product to define orthogonality, find the orthogonal projection of the vector $(0,0,1)$ onto the plane defined by the equation

$$x - y + z = 0$$

(F16 10:10 Exam 1) (Solutions)

[4] By least squares, find the equation of the form $y = ax^2 + b$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$$

(Note that x is *squared* in $y = ax^2 + b$.)

[5] Let V be the vector space $\mathbb{R}^3$, equipped with the inner product

$$\langle (a,b,c),(d,e,f) \rangle = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} d \\ e \\ f \end{bmatrix}$$

Using this inner product to define orthogonality, find an orthogonal basis for the plane defined by the equation

$$x + y = 0$$

Extend this basis to an orthogonal basis for $\mathbb{R}^3$.

(F16 8:40 Exam 2) (Solutions)

[1] Find the determinant of the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 & 3 \\ 0 & 1 & 1 & 5 & 5 \\ 2 & 2 & 2 & 2 & 2 \\ 0 & 1 & 1 & 1 & 7 \end{bmatrix}$$

[2] Find the inverse to the matrix

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

[3] Using Cramer's rule, solve for x in the system of equations

$$\begin{bmatrix} 1 & a & 0 \\ 2 & b & 1 \\ 4 & c & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$$

(F16 10:10 Exam 2) (Solutions)

[1] Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 1 & 5 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 3 & 0 & 0 & 0 \\ 1 & 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 1 & 4 \end{bmatrix}$$

[2] Find the inverse to the matrix

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

[3] Using Cramer's rule, solve for y in the system of equations

$$\begin{bmatrix} a & 1 & 2 \\ b & 1 & 1 \\ c & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$$

(F15 Homework 1) (Solutions)

[4] Find the matrix A such that

$$A \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 4 & 4 \\ 3 & 3 & 1 \end{bmatrix}$$

(F15 Homework 2)

[2] Find the 3×3 matrix that vanishes on the plane $3x - 2y + z = 0$, and maps the vector $(1, 0, 0)$ to itself.

[3] Find the 3×3 matrix that vanishes on the vector $(1, 0, 2)$, and stretches each vector in the plane $x + y = 0$ by a factor of 3.

[4] Find the 3×3 matrix that projects orthogonally onto the line

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} t$$

[5] Find the 3×3 matrix that projects orthogonally onto the plane

$$x - 2y + z = 0$$

[7] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$$

[8] Find an orthogonal basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors

$$(1, 2, 0, 0) \quad (0, 1, 2, 0) \quad (0, 0, 1, 2) \quad (1, 0, -4, 0) \quad (0, 1, 0, -4)$$

Extend this basis to an orthogonal basis for $\mathbb{R}^4$.

[9] Let V be the vector space of all polynomials of degree $\leqslant 2$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace consisting of those polynomials $f(x)$ such that $f(0) = 0$. Find the orthogonal projection of the polynomial $x + 2$ onto the subspace W, with respect to the inner product

$$\langle f, g \rangle = \int_0^1 f(x) g(x)\, dx$$

(F15 Exam 2) (Solutions)

[2] Find the 3×3 matrix A such that

$$A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad A \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \qquad A \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

[3] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 3 & 2 \end{bmatrix}$$

[4] Find the 4×4 matrix that projects orthogonally onto the plane spanned by the vectors $(1,0,1,0)$ and $(0,1,0,1)$.

[5] Let V be the vector space $\mathbb{R}^3$, equipped with the inner product

$$\langle (a,b,c),(d,e,f) \rangle \;=\; \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} d \\ e \\ f \end{bmatrix}$$

Using this inner product rather than the dot product, find the orthogonal projection of the vector $(1,0,0)$ onto the plane spanned by $(0,1,0)$ and $(0,0,1)$.

(F15 Homework 3)

[1] Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 1 & 3 & 3 & 1 \\ 1 & 4 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

[2] Find the determinant of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 & 1 & 1 \\ 1 & 3 & 1 & 2 & 1 \\ 5 & 1 & 4 & 1 & 1 \\ 1 & 2 & 1 & 3 & 1 \\ 3 & 3 & 3 & 3 & 3 \end{bmatrix}$$

[3] Find the inverse of the matrix

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

[4] Using Cramer's rule, solve for x in the system of equations

$$\begin{bmatrix} 3 & a & 2 \\ 2 & b & 3 \\ 1 & c & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

[5] Find the characteristic equation and a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 1 & 4 \\ 1 & -2 \end{bmatrix}$$

[7] Express $f(n)$ using a matrix power, and find $f(8)$, where

$$\begin{aligned} f(0) = -1, \quad f(1) &= 2 \\ f(n) &= f(n-1) + f(n-2) \end{aligned}$$

[8] Express $f(n)$ using a matrix power, and find $f(8)$, where

$$\begin{aligned} f(0) = 1, \quad f(1) = 1, \quad g(1) &= 1 \\ f(n) &= f(n-1) + g(n-1) \\ g(n) &= f(n-1) + f(n-2) \end{aligned}$$

[9] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[\,] \qquad \begin{bmatrix} 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Find $f(0)$ and $f(1)$. Find a recurrence relation for $f(n)$. Express $f(n)$ using a matrix power. Find $f(8)$.

(F15 Exam 3)

[1] Find the determinant of the matrix

$$A = \begin{bmatrix} 3 & 6 & 6 & 1 & 1 \\ 1 & 3 & 6 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 1 & 1 & 6 \\ 1 & 1 & 1 & 1 & 3 \end{bmatrix}$$

[2] Using Cramer's rule, solve for z in the system of equations

$$\begin{bmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

[3] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$\begin{bmatrix} 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 2 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 2 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Find $f(10)$.

[4] Find a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 4 & 6 \\ 1 & 5 \end{bmatrix}$$

(F15 Final) (Solutions)

[2] Find the 3×3 matrix A such that

$$A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad A \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad A \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

[3] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$\begin{bmatrix} 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Find $f(8)$.

(F14 Practice 1) (Solutions)

[4] Find the matrix A such that

$$A \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ -3 & 3 & 0 \\ 0 & -3 & 0 \end{bmatrix}$$

(F14 Homework 1) (Solutions)

[4] Find the matrix A such that

$$A \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 3 \\ 0 & 0 & 3 \\ 0 & 0 & 3 \end{bmatrix}$$

(F14 8:40 Exam 1) (Solutions)

[4] Find the matrix A such that

$$A \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

(F14 11:40 Exam 1) (Solutions)

[4] Find the matrix A such that

$$A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix}$$

(F14 Practice 2) (Solutions)

[2] Find the 3×3 matrix that vanishes on the plane $x + y + z = 0$, and maps the vector $(1, 0, 0)$ to itself.

[3] Find the 3×3 matrix that vanishes on the vector $(1, 1, 1)$, and maps each point on the plane $x + y = 0$ to itself.

[4] Find the 3×3 matrix that projects orthogonally onto the line

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} t$$

[5] Find the 3×3 matrix that projects orthogonally onto the plane

$$x + y + z = 0$$

[7] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 2 \end{bmatrix}$$

[8] Find an orthogonal basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors

$$(1, -1, 0, 0) \qquad (0, 1, -1, 0) \qquad (0, 0, 1, -1) \qquad (1, 0, -1, 0) \qquad (0, 1, 0, -1)$$

Extend this basis to an orthogonal basis for $\mathbb{R}^4$.

[9] Let V be the vector space of all polynomials of degree $\leqslant 2$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace consisting of those polynomials $f(x)$ such that $f(1) = 0$. Find the orthogonal projection of the polynomial x onto the subspace W, with respect to the inner product

$$\langle f, g \rangle = \int_0^1 f(x) g(x) \, dx$$

(F14 Homework 2) (Solutions)

[2] Find the 3×3 matrix that vanishes on the plane $4x + 2y + z = 0$, and maps the vector $(1, 1, 1)$ to itself.

[3] Find the 3×3 matrix that vanishes on the vector $(1, 1, 0)$, and maps each point on the plane $x + 2y + 2z = 0$ to itself.

[4] Find the 3×3 matrix that projects orthogonally onto the line

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} t$$

[5] Find the 3×3 matrix that projects orthogonally onto the plane

$$x + 2y + 3z = 0$$

[7] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 2 & 3 \end{bmatrix}$$

[8] Find an orthogonal basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors

$$(1,2,0,0) \qquad (0,1,2,0) \qquad (1,3,3,2) \qquad (0,0,1,2) \qquad (1,3,3,2)$$

Extend this basis to an orthogonal basis for $\mathbb{R}^4$.

[9] Let V be the vector space of all polynomials of degree $\leqslant 2$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace consisting of those polynomials $f(x)$ such that $f(-1) = 0$. Find the orthogonal projection of the polynomial $x+1$ onto the subspace W, with respect to the inner product

$$\langle f, g \rangle = \int_0^1 f(x)g(x)\, dx$$

(F14 8:40 Exam 2) (Solutions)

[1] Find the 3×3 matrix that maps the vector $(0,1,1)$ to $(0,2,2)$, and maps each point on the plane $x+y=0$ to the zero vector.

[3] Find the 3×3 matrix that projects orthogonally onto the plane

$$x + 2y = 0$$

[4] Find an orthogonal basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors

$$(1,-2,0,0) \qquad (1,0,-2,0) \qquad (1,0,0,-2) \qquad (0,1,-1,0) \qquad (0,1,0,-1) \qquad (0,0,1,-1)$$

Extend this basis to an orthogonal basis for $\mathbb{R}^4$.

[5] Let V be the vector space of all polynomials of degree $\leqslant 2$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace of V consisting of those polynomials $f(x)$ such that the second derivative $f''(x) = 0$.
Find the orthogonal projection of the polynomial x^2 onto the subspace W, with respect to the inner product

$$\langle f, g \rangle = \int_0^1 f(x)g(x)\, dx$$

(F14 11:40 Exam 2) (Solutions)

[1] Find the 3×3 matrix that maps the vector $(1,1,1)$ to $(2,2,2)$, and maps each point on the plane $x+y+z=0$ to itself.

[3] Find the 3×3 matrix that projects orthogonally onto the plane

$$x + y - 2z = 0$$

[4] Find an orthogonal basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors

$$(-1, 1, 0, -1) \qquad (-1, 0, 1, -1) \qquad (0, 1, -1, 0) \qquad (-2, 1, 1, -2) \qquad (1, 1, -2, 1)$$

Extend this basis to an orthogonal basis for $\mathbb{R}^4$.

[5] Let V be the vector space of all polynomials of degree $\leqslant 2$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace of V consisting of those polynomials $f(x)$ such that the derivative $f'(0) = 0$.
Find the orthogonal projection of the polynomial x onto the subspace W, with respect to the inner product

$$\langle f, g \rangle \;=\; \int_0^1 f(x) g(x)\, dx$$

(F14 Practice 3) (Solutions)

[1] Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 3 & 4 & 1 & 5 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

[2] Find the determinant of the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 0 \\ 1 & 1 & 3 & 1 & 2 \\ 1 & 1 & 1 & 3 & 0 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix}$$

[3] Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

[4] Using Cramer's rule, solve for x in the system of equations

$$\begin{bmatrix} 3 & 1 & a \\ 2 & 1 & b \\ 1 & 1 & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 2 \end{bmatrix}$$

[5] Find the characteristic equation and a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

[7] Express $f(n)$ using a matrix power, and find $f(8)$, where

$$\begin{aligned} f(0) = 1, \quad f(1) = 2 \\ f(n) \;=\; 2\,f(n-1) \;-\; f(n-2) \end{aligned}$$

[8] Express $f(n)$ using a matrix power, and find $f(8)$, where

$$\begin{aligned} f(0) = 1, \quad f(1) = 1, \quad g(1) = 2 \\ f(n) \;=\; f(n-1) \;+\; g(n-1) \;+\; f(n-2) \\ g(n) \;=\; f(n-1) \;-\; g(n-1) \;+\; f(n-2) \end{aligned}$$

[9] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[\,] \qquad [\,1\,] \qquad \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Find $f(0)$ and $f(1)$. Find a recurrence relation for $f(n)$. Express $f(n)$ using a matrix power. Find $f(8)$.

(F14 Homework 3) (Solutions)

[1] Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 1 & 3 & 3 & 1 \\ 1 & 4 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

[2] Find the determinant of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 & 1 & 1 \\ 1 & 3 & 1 & 2 & 1 \\ 5 & 1 & 4 & 1 & 1 \\ 1 & 2 & 1 & 3 & 1 \\ 3 & 3 & 3 & 3 & 3 \end{bmatrix}$$

[3] Find the inverse of the matrix

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

[4] Using Cramer's rule, solve for x in the system of equations

$$\begin{bmatrix} 3 & a & 2 \\ 2 & b & 3 \\ 1 & c & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

[5] Find the characteristic equation and a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 1 & 4 \\ 1 & -2 \end{bmatrix}$$

[7] Express $f(n)$ using a matrix power, and find $f(8)$, where

$$f(0) = -1, \quad f(1) = 2$$
$$f(n) = f(n-1) + f(n-2)$$

[8] Express $f(n)$ using a matrix power, and find $f(8)$, where

$$f(0) = 1, \quad f(1) = 1, \quad g(1) = 1$$
$$f(n) = f(n-1) + g(n-1)$$
$$g(n) = f(n-1) + f(n-2)$$

[9] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[\,] \qquad [\,1\,] \qquad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Find $f(0)$ and $f(1)$. Find a recurrence relation for $f(n)$. Express $f(n)$ using a matrix power. Find $f(8)$.

(F14 8:40 Exam 3) (Solutions)

[1] Find the determinant of the matrix

$$\begin{bmatrix} 4 & 6 & 2 & 1 \\ 2 & 1 & 2 & 1 \\ 1 & 6 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

[2] Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

[3] Using Cramer's rule, solve for z in the system of equations

$$\begin{bmatrix} 1 & a & 1 \\ 2 & b & 3 \\ 1 & c & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

[4] Find the characteristic equation and a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 0 & -1 \\ 3 & -4 \end{bmatrix}$$

[5] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[\,] \quad \begin{bmatrix} 2 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Find $f(0)$ and $f(1)$. Find a recurrence relation for $f(n)$. Express $f(n)$ using a matrix power. Find $f(8)$.

(F14 11:40 Exam 3) (Solutions)

[1] Find the determinant of the matrix

$$A = \begin{bmatrix} 3 & 3 & 3 & 3 \\ 1 & 4 & 1 & 1 \\ 1 & 2 & 1 & 4 \\ 2 & 2 & 4 & 2 \end{bmatrix}$$

[2] Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & 2 \end{bmatrix}$$

[3] Using Cramer's rule, solve for y in the system of equations

$$\begin{bmatrix} a & 1 & 2 \\ b & 1 & 3 \\ c & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

[4] Find the characteristic equation and a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 2 & -2 \\ -3 & 1 \end{bmatrix}$$

[5] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[\,] \quad \begin{bmatrix} -2 \end{bmatrix} \quad \begin{bmatrix} -2 & 1 \\ 2 & -2 \end{bmatrix} \quad \begin{bmatrix} -2 & 1 & 0 \\ 2 & -2 & 1 \\ 0 & 2 & -2 \end{bmatrix} \quad \begin{bmatrix} -2 & 1 & 0 & 0 \\ 2 & -2 & 1 & 0 \\ 0 & 2 & -2 & 1 \\ 0 & 0 & 2 & -2 \end{bmatrix} \quad \begin{bmatrix} -2 & 1 & 0 & 0 & 0 \\ 2 & -2 & 1 & 0 & 0 \\ 0 & 2 & -2 & 1 & 0 \\ 0 & 0 & 2 & -2 & 1 \\ 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

Find $f(0)$ and $f(1)$. Find a recurrence relation for $f(n)$. Express $f(n)$ using a matrix power. Find $f(8)$.

(F14 8:40 Final) (Solutions)

[2] Find the 3×3 matrix A that maps the vector $(1, 2, 1)$ to $(3, 6, 3)$, and maps each point on the plane $x+y+z = 0$ to the zero vector.

[3] Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

(F14 11:40 Final) (Solutions)

[2] Find the 3×3 matrix A that maps the vector $(1, 1, 0)$ to $(2, 2, 0)$, and maps each point on the plane $x+y+z = 0$ to itself.

[3] Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & 1 \\ 1 & 3 & 2 \end{bmatrix}$$

(S14 Exam 2) (Solutions)

[2] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$$

[3] Find the 3×3 matrix that projects orthogonally onto the plane

$$x + 3y - 2z \;=\; 0$$

[4] Find an orthogonal basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors

$$(1,1,0,0) \qquad (0,1,1,0) \qquad (0,0,1,1) \qquad (1,2,1,0) \qquad (0,1,2,1)$$

Extend this basis to an orthogonal basis for $\mathbb{R}^4$.

[5] Let V be the vector space of all polynomials of degree $\leqslant 2$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace of polynomials of degree $\leqslant 1$. Find the orthogonal projection of the polynomial x^2 onto the subspace W, with respect to the inner product

$$\langle f, g \rangle \;=\; \int_0^1 f(x)g(x)\,dx$$

(S14 Exam 3) (Solutions)

[1] Find the determinant of the matrix

$$\begin{bmatrix} 2 & 2 & 3 & 3 \\ 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 6 \\ 3 & 3 & 4 & 9 \end{bmatrix}$$

[2] Find the inverse of the matrix

$$\begin{bmatrix} 0 & 2 & 1 \\ 3 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix}$$

[3] Find w/y where

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

(S14 Final) (Solutions)

[2] Find the 3×3 matrix that projects orthogonally onto the plane

$$x - z = 0$$

(F13 Exam 2) (Solutions)

[2] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$(x_1, y_1) = (-1, 0), \quad (x_2, y_2) = (0, 0), \quad (x_3, y_3) = (1, 0), \quad (x_4, y_4) = (2, 1)$$

[3] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that projects orthogonally onto the subspace V spanned by $(1, -1, 0)$ and $(0, 2, 1)$. Find the matrix A that represents L in standard coordinates.

[4] Let V be the vector space of all polynomials of degree $\leqslant 2$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace of polynomials satisfying $f(2) = 0$. Find an orthogonal basis for W with respect to the inner product

$$\langle f, g \rangle = \int_0^1 f(x) g(x) \, dx$$

[5] Find an orthogonal basis for the subspace of $\mathbb{R}^4$ defined by the equation $w + x - 2y - 2z = 0$. Extend this basis to a orthogonal basis for $\mathbb{R}^4$.

(F13 Exam 3) (Solutions)

[1] Find the determinant of the matrix

$$\begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 4 & 1 & 1 \\ 2 & 3 & 3 & 2 \\ 1 & 4 & 2 & 3 \end{bmatrix}$$

[2] Find the determinant of the matrix

$$\begin{bmatrix} 2 & 3 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

[3] Find x/y where

$$\begin{bmatrix} a & b & c & d \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

[4] Find the inverse of the matrix

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 0 \\ 1 & 3 & 0 \end{bmatrix}$$

(F13 Final) (Solutions)

[2] Find an orthogonal basis for the subspace of $\mathbb{R}^4$ defined by the equation $w + x - y - z = 0$. Extend this basis to a orthogonal basis for $\mathbb{R}^4$.

[3] Find the determinant of the matrix

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 \end{bmatrix}$$

(S13 8:40 Exam 2) (Solutions)

[2] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$(x_1, y_1) = (0, 0), \quad (x_2, y_2) = (1, 0), \quad (x_3, y_3) = (3, 1)$$

[3] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that projects orthogonally onto the line

$$x = y = 2z$$

Find the matrix A that represents L in standard coordinates.

[4] Find an orthogonal basis for the subspace of $\mathbb{R}^4$ given by the equation $w + x + y - 2z = 0$.

[5] Let V be the vector space of all polynomials of degree $\leqslant 3$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace of polynomials satisfying $f(0) = f(1) = 0$. Find an orthogonal basis for W with respect to the inner product

$$\langle f, g \rangle = \int_0^1 f(x)g(x)\, dx$$

(S13 10:10 Exam 2) (Solutions)

[2] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$(x_1, y_1) = (0, 1), \quad (x_2, y_2) = (1, 0), \quad (x_3, y_3) = (2, 2)$$

[3] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that projects orthogonally onto the subspace

$$x + 2y + z = 0$$

Find the matrix A that represents L in standard coordinates.

[4] Find an orthogonal basis for the subspace of $\mathbb{R}^4$ spanned by the vectors

$$(1, 1, 1, 1), \quad (1, 2, 1, 2), \quad (2, 1, 2, 1), \quad (2, 2, 2, 2)$$

[5] Let V be the vector space of all polynomials of degree $\leqslant 3$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace of polynomials satisfying $f(0) = f'(0) = 0$. Find an orthogonal basis for W with respect to the inner product

$$\langle f, g \rangle \ = \ \int_0^1 f(x)g(x)\, dx$$

(S13 Alt Exam 2) (Solutions)

[2] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$(x_1, y_1) = (-1, 0), \quad (x_2, y_2) = (0, 0), \quad (x_3, y_3) = (1, 2)$$

[4] Find an orthogonal basis for the subspace of $\mathbb{R}^4$ spanned by the vectors

$$(1, 1, 1, 1), \quad (1, 2, 2, 2), \quad (1, 1, 2, 2), \quad (3, 4, 5, 5)$$

[5] Let V be the vector space of all polynomials of degree $\leqslant 4$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace of odd polynomials: those polynomials such that $f(-x) = -f(x)$. Find an orthogonal basis for W with respect to the inner product

$$\langle f, g \rangle \ = \ \int_0^1 f(x)g(1-x)\, dx$$

(S13 8:40 Exam 3) (Solutions)

[1] Compute the determinant of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 4 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 9 & 1 \end{bmatrix}$$

[2] Find w/z where

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

(S13 10:10 Exam 3) (Solutions)

[1] Compute the determinant of the matrix

$$A = \begin{bmatrix} 0 & 3 & 2 & 0 \\ 3 & 6 & 9 & 2 \\ 2 & 9 & 6 & 3 \\ 0 & 2 & 3 & 0 \end{bmatrix}$$

[2] Find w/z where

$$\begin{bmatrix} a & b & c & d \\ 1 & 1 & 5 & 1 \\ 1 & 0 & 1 & 1 \\ 3 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(S13 Alt Exam 3) (Solutions)

[1] Compute the determinant of the matrix

$$A = \begin{bmatrix} 0 & 2 & 1 & 0 \\ 1 & 3 & 4 & 0 \\ 0 & 4 & 3 & 1 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

[2] Find w/z where

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

(S13 8:40 Final)

[2] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that projects orthogonally onto the plane

$$x - y = 0$$

Find the matrix A that represents L in standard coordinates.

[3] Let V be the vector space of all polynomials $f(x)$ of degree $\leqslant 2$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace of V consisting of those polynomials satisfying $f(1) = f(-1)$. Find an orthogonal basis for W with respect to the inner product

$$\langle f, g \rangle \;=\; \int_0^1 f(x)g(x)\,dx$$

(S13 10:10 Final)

[2] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that projects orthogonally onto the line

$$x = y = z$$

Find the matrix A that represents L in standard coordinates.

[3] Let V be the vector space of all polynomials $f(x)$ of degree $\leqslant 2$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace of V consisting of those polynomials satisfying $f(0) = 0$. Find an orthogonal basis for W with respect to the inner product

$$\langle f, g \rangle \;=\; \int_0^1 f(x)g(x)\,dx$$

(S13 Alt Final)

[2] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that projects orthogonally onto the plane

$$x + y + z = 0$$

Find the matrix A that represents L in standard coordinates.

[3] Let V be the vector space of all polynomials $f(x)$ of degree $\leqslant 2$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace of V consisting of those polynomials satisfying $f(0) = f(1)$. Find an orthogonal basis for W with respect to the inner product

$$\langle f, g \rangle \;=\; \int_0^1 f(x)g(x)\,dx$$

(S12 Practice 2) (Solutions)

[7] Let V be the subspace of $\mathbb{R}^4$ consisting of all solutions to the system of equations

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Let W be the orthogonal complement of V. Find vectors $v \in V$ and $w \in W$ so $v + w = (1, 0, 1, 0)$.

[8] Let V be the subspace of $\mathbb{R}^4$ consisting of all solutions to the system of equations

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Let W be the orthogonal complement of V. Find vectors $v \in V$ and $w \in W$ so $v + w = (0, 0, 1, 1)$.

[9] Let V be the subspace of $\mathbb{R}^4$ consisting of all solutions to the system of equations

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Let W be the orthogonal complement of V. Find vectors $v \in V$ and $w \in W$ so $v + w = (1, 0, 0, 0)$.

(S12 9:10 Exam 2) (Solutions)

[1] Let V be the vector space of all polynomials $f(x)$ of degree $\leqslant 2$. Find a basis for the subspace W defined by

$$f(1) = 0.$$

Extend this to a basis for V.

[2] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 1 & 3 \\ 2 & 0 \end{bmatrix}$$

[3] Let V be the subspace of $\mathbb{R}^5$ spanned by the vectors

$$(1, 1, 1, 0, 0), \quad (0, 0, 1, 1, 1).$$

Find an orthogonal basis for the subspace V.

[4] Using Cramer's rule, solve for z in

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 3 & 0 & 1 \\ 1 & 3 & 3 & 1 \\ 1 & 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

[5] Let V be the subspace of $\mathbb{R}^4$ consisting of all solutions to the system of equations

$$\begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Find the matrix A that projects orthogonally onto the subspace V.

(S12 11:00 Exam 2) (Solutions)

[1] Let V be the vector space of all polynomials $f(x)$ of degree $\leqslant 2$. Find a basis for the subspace W defined by

$$f'(1) = 0.$$

Extend this to a basis for V.

[2] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 3 \\ 2 & 2 \end{bmatrix}$$

[3] Let V be the subspace of $\mathbb{R}^4$ spanned by the vectors

$$(1, 1, 1, 2), \quad (2, 1, 1, 1).$$

Find an orthogonal basis for the subspace V.

[4] Find the determinant of the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 & a & 0 \\ 1 & b & 0 & 0 & 0 \\ 0 & 0 & 1 & c & 0 \\ 1 & d & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e \end{bmatrix}.$$

[5] Let V be the subspace of $\mathbb{R}^4$ spanned by the vectors

$$(1,1,1,1), \quad (1,1,2,2), \quad (1,1,3,3).$$

Find the matrix A that projects orthogonally onto the subspace V.

(S11 Exam 2) (Solutions)

[2] Find the determinant of each of the following matrices.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 3 & 4 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 9 \end{bmatrix} \qquad \begin{bmatrix} a & b & c & d \\ a & b+1 & c & d \\ a & b & c+1 & d \\ a & b & c & d+1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 4 \\ 1 & 2 & 6 & 4 \\ 1 & 2 & 3 & 8 \end{bmatrix}$$

[3] Find the inverse of the following matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ a & 1 & 0 & c \\ b & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[4] Let

$$v_1 = (1,0,0), \qquad v_2 = (1,1,0), \qquad v_3 = (0,1,1)$$

Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear map such that

$$L(v_1) = v_2, \qquad L(v_2) = v_3, \qquad L(v_3) = v_1,$$

Find the matrix A (in standard coordinates) that represents the linear map L.

[5] Find the ratio x/y for the solution to the matrix equation

$$\begin{bmatrix} a & d & 1 \\ b & e & 1 \\ c & f & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

[6] Find the determinant of the following 5×5 matrix. What is the determinant for the $n \times n$ case?

$$\begin{bmatrix} x & x^2 & 0 & 0 & 0 \\ 1 & x & x^2 & 0 & 0 \\ 0 & 1 & x & x^2 & 0 \\ 0 & 0 & 1 & x & x^2 \\ 0 & 0 & 0 & 1 & x \end{bmatrix}$$

(S11 Final) (Solutions)

[1] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} \quad = \quad \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 3 & 2 \end{bmatrix}$$

[2] Extend the vector $(1, 1, 1, 2)$ to an orthogonal basis for $\mathbb{R}^4$.

[3] Find the orthogonal projection of the vector $(1, 0, 0, 0)$ onto the subspace of $\mathbb{R}^4$ spanned by the vectors $(1, 1, 1, 0)$ and $(0, 1, 1, 1)$.

[4] Find the matrix A that projects $\mathbb{R}^4$ orthogonally onto the subspace spanned by the vectors $(1, 1, 1, 1)$ and $(1, 1, 2, 2)$.

[5] Find the eigenvalues and corresponding eigenvectors of the matrix

$$A \quad = \quad \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$$