F17 10:10 Exam 2 Problem 1
Linear Algebra, Dave Bayer

test1b2p1

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Test 1

Name Answer Key Uni

[1] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}$$

$y = \boxed{1/10}\, x + \boxed{7/10}$

$$\begin{pmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 6 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{3}{10} \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{10} \\ \frac{7}{10} \end{pmatrix}$$

check:

x	y	ax+b	Δ×10
-1	1	6/10	-4
0	0	7/10	7
1	1	8/10	-2
2	1	9/10	-1

⊥ to $\begin{matrix} 1 \\ 1 \\ 1 \\ 1 \end{matrix}$, $\begin{matrix} -1 \\ 0 \\ 1 \\ 2 \end{matrix}$

✓ ✓

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Test 1

[2] Find the inverse of the matrix

$$A = \begin{bmatrix} 5 & 2 & 1 \\ 3 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\boxed{5}} \begin{bmatrix} \boxed{1} & \boxed{1} & \boxed{-3} \\ \boxed{1} & \boxed{-4} & \boxed{7} \\ \boxed{-2} & \boxed{3} & \boxed{1} \end{bmatrix}$$

(Do not write a negative denominator.)

$$\texttt{Inverse}\left[\begin{pmatrix} 5 & 2 & 1 \\ 3 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}\right] = \begin{pmatrix} 1 & 1 & -3 \\ 1 & -4 & 7 \\ -2 & 3 & 1 \end{pmatrix} / 5$$

$$\begin{array}{c|cccc|} & 5 & 3 & 1 & 5 \; 3 \\ \hline 2 & 1 & 1 & 2 & 1 \\ 1 & 2 & 1 & 1 & 2 \\ 5 & 3 & 1 & 5 & 3 \\ 2 & 1 & 1 & 2 & 1 \\ \hline \end{array}$$

$$\begin{bmatrix} -1 & -1 & 3 \\ -1 & 4 & -7 \\ 2 & -3 & -1 \end{bmatrix} / -5 \quad \text{so fix signs}$$

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Test 1

[3] Consider $\mathbb{R}^3$ equipped with the inner product

$$\langle (a,b,c),(d,e,f)\rangle \;=\; \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} d \\ e \\ f \end{bmatrix}$$

Using this inner product, find the orthogonal projection of the vector $(3,3,3)$ onto the plane spanned by $(1,0,0)$ and $(0,1,0)$.

$$\begin{bmatrix} 2 & 5 & 0 \end{bmatrix}$$

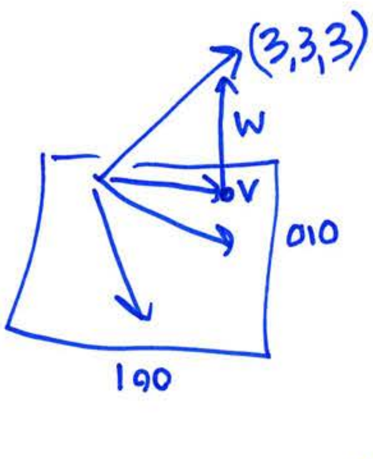
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \end{pmatrix}$$

$$\begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}$$

$A^TAx = A^Tb$ solves usual projection
$A^TBAx = A^TBb$ solves for inner product using B



$(3,3,3) = v + w$ v in plane, answer
w $\perp$ to plane

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \quad \Big\} \text{ find } w \perp \text{ to plane}$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0 \quad \text{so } w = t(1,-2,3)$$

want $(3,3,3) - t(1,-2,3)$ in plane
$\Rightarrow t = 1$ (so last coord zero)
$\Rightarrow \boxed{v = (2,5,0)}$

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Test 1

[4] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$\begin{bmatrix} 2 \end{bmatrix} \qquad \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix} \qquad \begin{bmatrix} 2 & 5 & 0 \\ 1 & 2 & 5 \\ 0 & 1 & 2 \end{bmatrix} \qquad \begin{bmatrix} 2 & 5 & 0 & 0 \\ 1 & 2 & 5 & 0 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Find $f(1)$ and $f(2)$. Find a recurrence relation for $f(n)$. Find $f(6)$.

$$f(1) = \boxed{2} \qquad f(2) = \boxed{-1}$$

$$f(n) = \left(\boxed{2}\right) f(n-1) + \left(\boxed{-5}\right) f(n-2)$$

$$f(6) = \boxed{139}$$

$$\text{Det}[(2)] = 2$$

$$\text{Det}\left[\begin{pmatrix} 2 & 5 \\ 1 & 2 \end{pmatrix}\right] = -1$$

$$\text{Det}\left[\begin{pmatrix} 2 & 5 & 0 \\ 1 & 2 & 5 \\ 0 & 1 & 2 \end{pmatrix}\right] = -12$$

$$f(n) = (2) f(n-1) + (-5) f(n-2)$$

$$\text{Det}\left[\begin{pmatrix} 2 & 5 & 0 & 0 & 0 & 0 \\ 1 & 2 & 5 & 0 & 0 & 0 \\ 0 & 1 & 2 & 5 & 0 & 0 \\ 0 & 0 & 1 & 2 & 5 & 0 \\ 0 & 0 & 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}\right] = 139$$

n	f(n)		
1	2	×(-5)	
2	-1	×2	×(-5)
3	-12		×2
4	-19		
5	22		
6	$\boxed{139}$		

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Test 1

[5] Find a system of eigenvalues and eigenvectors for the matrix A, where

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\lambda_1,\ \lambda_2 = 2,\ 5$$

$$v_1,\ v_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix},\ \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$$

$$10 - 7x + x^2 = 0$$

$$\begin{pmatrix} 5 & 2 \\ \{2, 1\} & \{-1, 1\} \end{pmatrix}$$

Check:

$$\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix} = 2\begin{bmatrix} -1 \\ 1 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix} = 5\begin{bmatrix} 2 \\ 1 \end{bmatrix} \checkmark$$