Linear Algebra, Dave Bayer

[1] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}$$

[2] Find the inverse of the matrix

$$A = \begin{bmatrix} 5 & 2 & 1 \\ 3 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

[3] Consider $\mathbb{R}^3$ equipped with the inner product

$$\langle (a,b,c),(d,e,f) \rangle \;=\; \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} d \\ e \\ f \end{bmatrix}$$

Using this inner product, find the orthogonal projection of the vector $(3,3,3)$ onto the plane spanned by $(1,0,0)$ and $(0,1,0)$.

[4] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$\begin{bmatrix} 2 \end{bmatrix} \qquad \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix} \qquad \begin{bmatrix} 2 & 5 & 0 \\ 1 & 2 & 5 \\ 0 & 1 & 2 \end{bmatrix} \qquad \begin{bmatrix} 2 & 5 & 0 & 0 \\ 1 & 2 & 5 & 0 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Find $f(1)$ and $f(2)$. Find a recurrence relation for $f(n)$. Find $f(6)$.

[5] Find a system of eigenvalues and eigenvectors for the matrix A, where

$$A \;=\; \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$