Our first exam will be held in class on Tuesday, October 3, 2017. Makeup exams will only be given under exceptional circumstances, and require a note from a doctor or a dean.

Exam 1 will consist of five questions. The following are the skills that one needs to learn for this exam:

- Use matrix multiplication to count paths in a graph.
- Find the general solution to a system of linear equations, expressed as a particular solution plus the set of all homogenous solutions.
- Express a matrix as a product of elementary matrices.
- Use Gaussian elimination to find the inverse of a matrix.
- Find the matrix or the set of matrices determined by a set of conditions, such as the effect on a basis, or a description as a projection.
- Find the row space and the column space of a matrix.
- Trim a set of vectors to an independent set. Extend an independent set of vectors to a basis.
- Find a basis for a subspace of $\mathbb{R}^n$. Extend this independent set to a basis for $\mathbb{R}^n$.

This material is covered in the first four chapters of Bretscher, and in past exam problems. You are encouraged to read the chapters in Bretscher carefully.

In addition, the following problem types were introduced in
(Practice1-LinearAlgebra-S12)
(Practice1-Solutions-LinearAlgebra-S12)

- Given a parametrization of an affine subspace of $\mathbb{R}^n$, find a system of linear equations having this affine subspace as its general solution.
- Find a parametrization for the intersection of two affine subspaces of $\mathbb{R}^n$, given either by parametrizations or by systems of equations.

Homework will count as 10% of your course grade. The homework for Exam 1 is given below. You may turn in problems in batchs as you complete them; homework that is received early will receive more careful feedback. All homework must be submitted on or before the day of our exam. There is a homework box on the fourth floor of the Mathematics building for your section of Linear Algebra. Please turn in homework to the box corresponding to your section. Please write your uni very clearly on each page of homework.

Please hand in the following problems; they are the same in both the 5e and 4e editions of Bretscher. (You are encouraged to work similar problems for your own practice.)

- 1.1 [10], 1.2 [10], 1.3 [24]
- 2.1 [14], 2.2 [10], 2.3 [16], 2.4 [20]
- 3.1 [24], 3.2 [32], 3.3 [20], 3.4 [24]
- 4.1 [20], 4.2 [14], 4.3 [2]

What follows on the remaining pages of this study guide are practice problems for our first exam, taken from past semesters of the course.

The sources for the following problems, along with many solutions, can be found on our Linear Algebra Course Materials page:

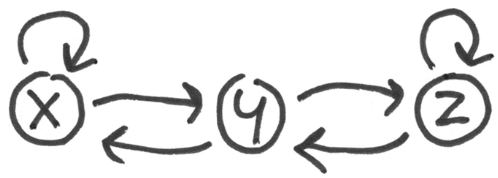
> https://www.math.columbia.edu/~bayer/LinearAlgebra/

(F16 8:40 Exam 1) (Solutions)

[1] Find the general solution to the following system of equations.

$$\begin{bmatrix} 2 & 1 & 1 & 3 \\ 1 & 1 & 0 & 1 \\ 3 & 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

[2] Using matrix multiplication, count the number of paths of length eight from y to y.



[3] Find the intersection of the following two affine subspaces of $\mathbb{R}^3$.

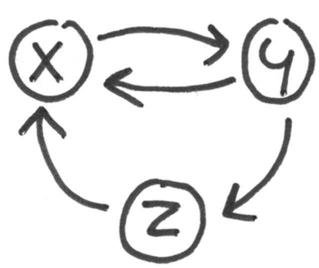
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}, \qquad x - 2y + z = 2$$

(F16 10:10 Exam 1) (Solutions)

[1] Find the general solution to the following system of equations.

$$\begin{bmatrix} 5 & 7 & 1 & 2 \\ 3 & 4 & 1 & 1 \\ 2 & 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

[2] Using matrix multiplication, count the number of paths of length 16 from x to x.



[3] Find the intersection of the following two affine subspaces of $\mathbb{R}^3$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}, \qquad\qquad 2x + y - z = 2$$

(F16 8:40 Final) (Solutions)

[1] Find the intersection of the following two affine subspaces of $\mathbb{R}^3$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} t \qquad\qquad x + z = 3$$

(F16 10:10 Final) (Solutions)

[1] Find the intersection of the following two affine subspaces of $\mathbb{R}^3$.

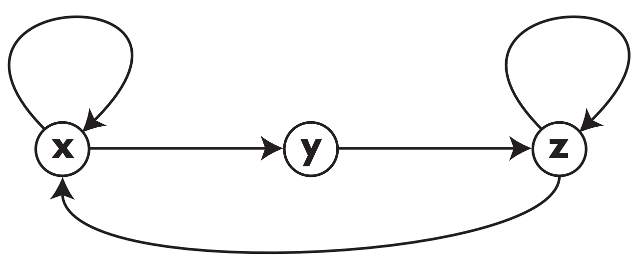
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} s \qquad\qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} t$$

(F15 Homework 1) (Solutions)

[1] Solve the following system of equations.

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 3 & 3 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

[2] Using matrix multiplication, count the number of paths of length nine from x to z.



[3] Express A as a product of three elementary matrices, where

$$A = \begin{bmatrix} 7 & 1 \\ 4 & 0 \end{bmatrix}$$

[5] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}$$

[6] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -2 & 1 \\ 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}$$

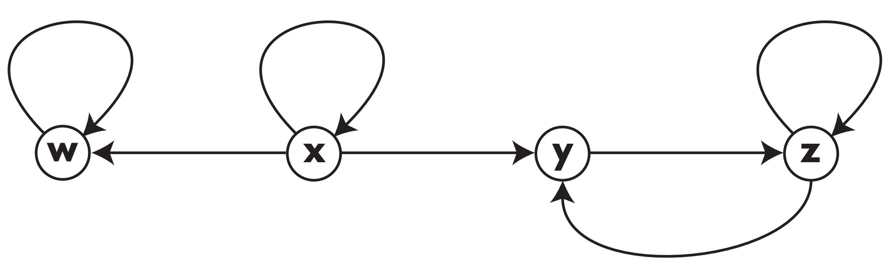
$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ -3 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} t \\ u \end{bmatrix}$$

(F15 Exam 1) (Solutions)

[1] Solve the following system of equations.

$$\begin{bmatrix} 0 & 1 & 1 & 2 & 3 \\ 1 & 1 & 2 & 3 & 5 \\ 1 & 2 & 3 & 5 & 8 \end{bmatrix} \begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

[2] Using matrix multiplication, count the number of paths of length ten from x to z.



[3] Express A as a product of four elementary matrices, where

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

[4] Find all 2×2 matrices A such that

$$A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

[5] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} r \\ s \\ t \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix} u$$

(F15 Homework 2)

[1] Let $v_1, v_2, \ldots v_r$ be a set of vectors that spans a subspace W of a vector space V. Prove that one can choose a subset of these vectors that forms a basis for W, and that this basis can be extended to a basis for V. Demonstrate this procedure on the vectors

$$(1,-1,0,0) \quad (1,0,-1,0) \quad (1,0,0,-1) \quad (0,1,-1,0) \quad (0,1,0,-1) \quad (0,0,1,-1)$$

[6] Find the row space and the column space of the matrix

$$\begin{bmatrix} 0 & 1 & 1 & 2 & 3 \\ 1 & 1 & 2 & 3 & 5 \\ 1 & 2 & 3 & 5 & 8 \end{bmatrix}$$

(F15 Exam 2) (Solutions)

[1] Find a basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors

$$(1,1,0,1) \qquad (1,0,1,1) \qquad (2,1,1,2) \qquad (3,2,1,3) \qquad (3,1,2,3) \qquad (4,2,2,4)$$

Extend this basis to a basis for $\mathbb{R}^4$.

(F15 Final) (Solutions)

[1] Solve the following system of equations.

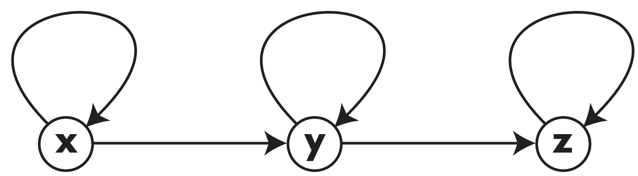
$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

(F14 Practice 1) (Solutions)

[1] Solve the following system of equations.

$$\begin{bmatrix} 6 & 1 & 8 & 1 \\ 4 & 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

[2] Using matrix multiplication, count the number of paths of length ten from x to z.



[3] Express A as a product of elementary matrices, where

$$A = \begin{bmatrix} -3 & 5 \\ 1 & 0 \end{bmatrix}$$

[5] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

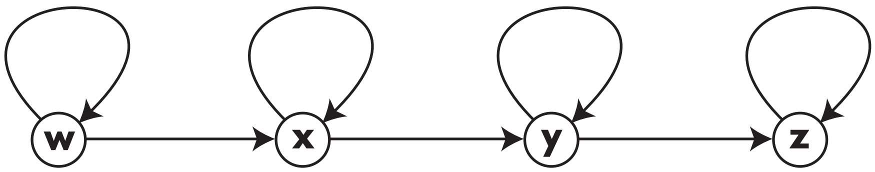
$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -2 & 1 \\ 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}$$

(F14 Homework 1) (Solutions)

[1] Solve the following system of equations.

$$\begin{bmatrix} 2 & -3 & -1 & 1 \\ 1 & -2 & -2 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

[2] Using matrix multiplication, count the number of paths of length ten from w to y.



[3] Express A as a product of elementary matrices, where

$$A = \begin{bmatrix} 0 & 1 & 12 \\ 1 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

[5] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

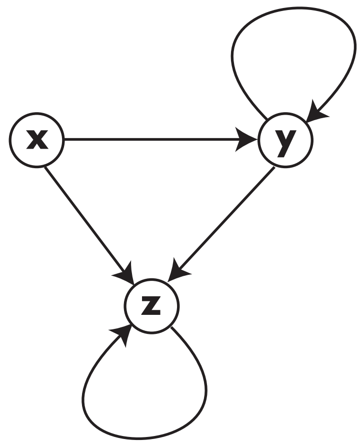
$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}$$

(F14 8:40 Exam 1) (Solutions)

[1] Solve the following system of equations.

$$\begin{bmatrix} 1 & 4 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$$

[2] Using matrix multiplication, count the number of paths of length eight from x to z.



[3] Express A as a product of elementary matrices, where

$$A \;=\; \begin{bmatrix} 0 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[5] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} \;=\; \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

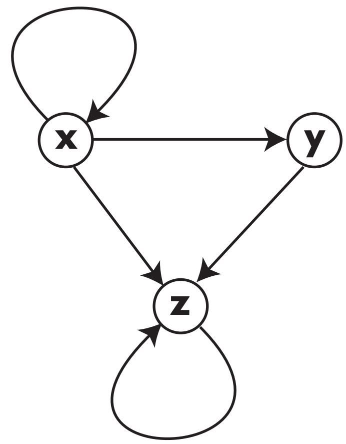
$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} \;=\; \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}$$

(F14 11:40 Exam 1) (Solutions)

[1] Solve the following system of equations.

$$\begin{bmatrix} 3 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 4 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} \;=\; \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$$

[2] Using matrix multiplication, count the number of paths of length eight from x to z.



[3] Express A as a product of elementary matrices, where

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[5] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} 1 & 1 & 1 & -2 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}$$

(F14 Practice 2) (Solutions)

[1] Find the 2×2 matrix that reflects across the line $x - 2y = 0$.

[6] Find the row space and the column space of the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 2 & 3 & 3 & 3 & 3 \end{bmatrix}$$

(F14 Homework 2) (Solutions)

[1] Find the 2×2 matrix that reflects across the line $3x - y = 0$.

[6] Find the row space and the column space of the matrix

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 & 6 & 10 \end{bmatrix}$$

(F14 8:40 Exam 2) (Solutions)

[2] Find a basis for the row space and a basis for the column space of the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ -2 & 0 & 0 & 1 & 1 & 0 \\ 0 & -2 & 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 0 & -1 & -1 \end{bmatrix}$$

(F14 11:40 Exam 2) (Solutions)

[2] Find a basis for the row space and a basis for the column space of the matrix

$$\begin{bmatrix} -1 & -1 & 0 & -2 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 & -2 \\ -1 & -1 & 0 & -2 & 1 \end{bmatrix}$$

(F14 8:40 Final) (Solutions)

[1] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}$$

(F14 11:40 Final) (Solutions)

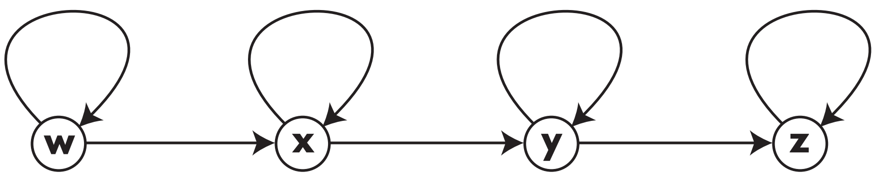
[1] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}$$

(S14 Exam 1) (Solutions)

[1] Using matrix multiplication, count the number of paths of length six from w to z.



[2] Solve the following system of equations.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 0 \\ 3 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}$$

[3] Express A as a product of elementary matrices, where

$$A = \begin{bmatrix} 4 & 3 & 6 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

[4] Find a system of equations having as solution set the following affine subspace of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

[5] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix}$$

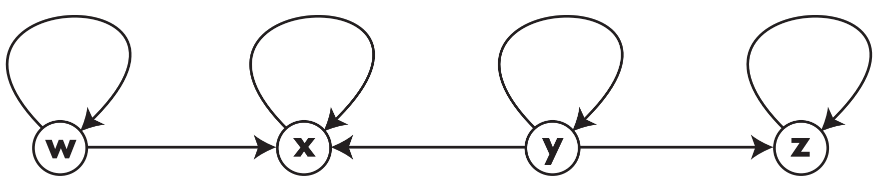
(S14 Exam 2) (Solutions)

[1] Find the row space and the column space of the matrix

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 6 & 8 \\ 0 & 3 & 6 & 9 & 2 \\ 0 & 4 & 8 & 2 & 6 \end{bmatrix}$$

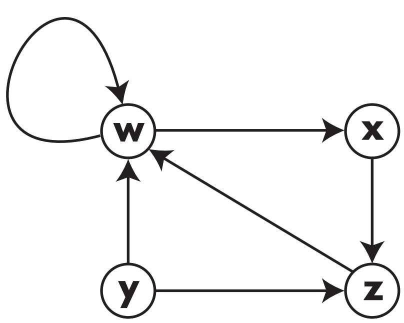
(S14 Final) (Solutions)

[1] Using matrix multiplication, count the number of paths of length ten from y to z.



(F13 Exam 1) (Solutions)

[1] Using matrix multiplication, count the number of paths of length ten from w to itself.



[2] Solve the following system of equations.

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 0 & 0 & 3 \\ 1 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$$

[3] Express A as a product of elementary matrices, where

$$A = \begin{bmatrix} 6 & 3 \\ 1 & 0 \end{bmatrix}$$

[4] Find a system of equations having as solution set the following affine subspace of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

[5] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix}$$

(F13 Exam 2) (Solutions)

[1] Find a basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors

$$(2, 0, 1, 0), \ (2, 0, 0, 1), \ (0, 2, 1, 0), \ (0, 2, 0, 1)$$

Extend this basis to a basis for $\mathbb{R}^4$.

(F13 Final) (Solutions)

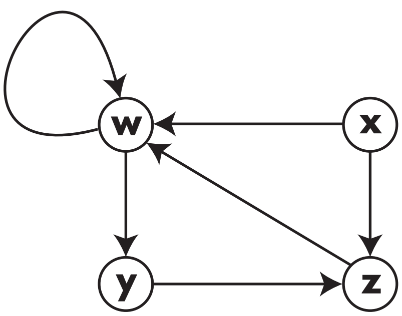
[1] Find the intersection of the following two affine subspaces of $\mathbb{R}^3$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix}$$

(S13 8:40 Exam 1) (Solutions)

[1] Using matrix multiplication, count the number of paths of length eight from w to itself.



[2] Solve the following system of equations.

$$\begin{bmatrix} 2 & 0 & 1 & 4 \\ 3 & 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$$

[3] Express A as a product of elementary matrices, where

$$A = \begin{bmatrix} -6 & 1 \\ 3 & 0 \end{bmatrix}$$

[4] Find a system of equations having as solution set the following affine subspace of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 & -2 \\ 3 & 3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

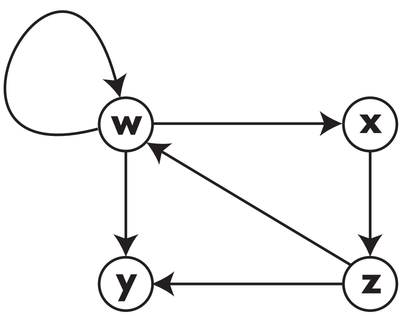
[5] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

(S13 10:10 Exam 1) (Solutions)

[1] Using matrix multiplication, count the number of paths of length eight from z to itself.



[2] Solve the following system of equations.

$$\begin{bmatrix} 2 & 3 & 1 & 0 \\ 6 & 8 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

[3] Express A as a product of elementary matrices, where

$$A = \begin{bmatrix} 0 & 1 \\ -4 & 3 \end{bmatrix}$$

[4] Find a system of equations having as solution set the following affine subspace of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ -4 & 5 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

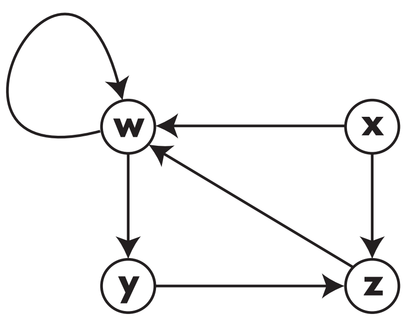
[5] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

(S13 Alt Exam 1) (Solutions)

[1] Using matrix multiplication, count the number of paths of length nine from y to itself.



[2] Solve the following system of equations.

$$\begin{bmatrix} 2 & 4 & 0 & 1 \\ 3 & 5 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

[3] Express A as a product of elementary matrices, where

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

[4] Find a system of equations having as solution set the following affine subspace of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

[5] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix}$$

(S13 8:40 Exam 2) (Solutions)

[1] Find a basis for the set of solutions to the system of equations

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 2 & 2 & 2 & 2 \\ 1 & 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Extend this basis to a basis for $\mathbb{R}^4$.

(S13 10:10 Exam 2) (Solutions)

[1] Find a basis for the set of solutions to the system of equations

$$\begin{bmatrix} 1 & 1 & 1 & 2 & 0 \\ 2 & 2 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Extend this basis to a basis for $\mathbb{R}^5$.

(S13 Alt Exam 2) (Solutions)

[1] Find a basis for the set of solutions to the system of equations

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 2 & 2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Extend this basis to a basis for $\mathbb{R}^4$.

[3] Let L be the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ that reflects first across the line $y = x$, then across the line $y = 3x$. Find the matrix A that represents L in standard coordinates.

(S13 8:40 Final)

[1] Find a system of equations having as solution set the following affine subspace of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} r \\ s \\ t \end{bmatrix}$$

(S13 10:10 Final)

[1] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

(S13 Alt Final)

[1] Find a system of equations having as solution set the following affine subspace of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 3 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

(S12 Practice 1) (Solutions)

[1] Find a system of equations having as solution set the following affine subspace of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 3 \\ 4 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 5 \\ 6 \\ 0 \\ 1 \end{bmatrix}$$

[2] Find a system of equations having as solution set the following affine subspace of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} + t \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

[3] Find a system of equations having as solution set the following affine subspace of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

[4] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

[5] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \end{bmatrix} + s \begin{bmatrix} -2 \\ 3 \\ 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 3 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

[6] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

[7] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \end{bmatrix} + s \begin{bmatrix} -2 \\ 3 \\ 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 3 \\ -2 \end{bmatrix}$$

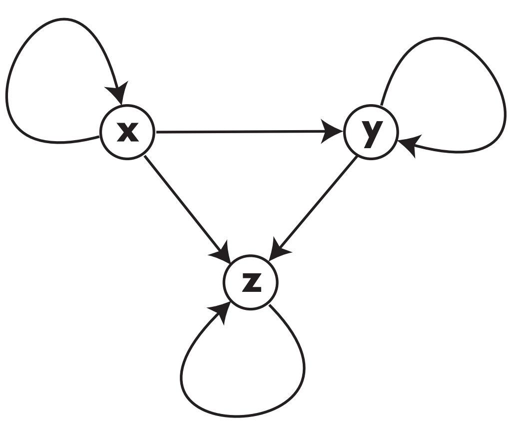
$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

(S12 9:10 Exam 1) (Solutions)

[1] Solve the following system of equations.

$$\begin{bmatrix} 1 & -1 & 0 & 1 \\ 1 & 0 & -1 & 1 \\ 3 & -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

[2] Using matrix multiplication, count the number of paths of length three from x to z.



[3] Express A as a product of elementary matrices, where

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}.$$

[4] Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that $L(v) = v$ for all v on the line $x + 2y = 0$, and $L(v) = 2v$ for all v on the line $x = y$. Find a matrix A that represents L in standard coordinates.

[5] Find a basis for the subspace V of $\mathbb{R}^4$ given by the equation $w + x + y + 2z = 0$. Extend this basis to a basis for all of $\mathbb{R}^4$.

[6] Find a system of equations having as solution set the following affine subspace of $\mathbb{R}^4$.

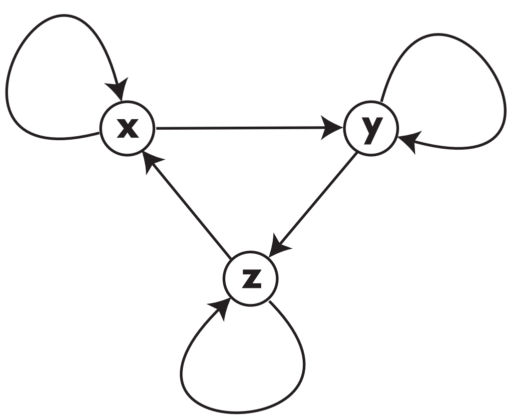
$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

(S12 11:00 Exam 1) (Solutions)

[1] Solve the following system of equations.

$$\begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 2 & -1 & -2 \\ 2 & 3 & -2 & -3 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

[2] Using matrix multiplication, count the number of paths of length three from x to z.



[3] Express A as a product of elementary matrices, where

$$A = \begin{bmatrix} 2 & 0 \\ 2 & 1 \end{bmatrix}.$$

[4] Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first reflects across the x-axis, and then reflects across the line $y = x$. Find a matrix A that represents L in standard coordinates.

[5] Find a basis for the subspace V of $\mathbb{R}^4$ spanned by the rows of the matrix

$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 2 \\ 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 6 \end{bmatrix}.$$

Extend this basis to a basis for all of $\mathbb{R}^4$.

[6] Find the intersection of the following two affine subspaces of $\mathbb{R}^3$.

$$\begin{bmatrix} 2 & -1 & 0 \\ 0 & 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

(S12 Practice 2) (Solutions)

[1] Let V and W be the subspaces of $\mathbb{R}^2$ spanned by $(1, 1)$ and $(1, 2)$, respectively. Find vectors $v \in V$ and $w \in W$ so $v + w = (2, -1)$.

[2] Let V and W be the subspaces of $\mathbb{R}^2$ spanned by $(1, -1)$ and $(2, 1)$, respectively. Find vectors $v \in V$ and $w \in W$ so $v + w = (1, 1)$.

[3] Let V and W be the subspaces of $\mathbb{R}^2$ spanned by $(1, 1)$ and $(1, 4)$, respectively. Find vectors $v \in V$ and $w \in W$ so $v + w = (2, 3)$.

[4] Let V be the subspace of $\mathbb{R}^3$ consisting of all solutions to the equation $x + y + z = 0$. Let W be the subspace of $\mathbb{R}^3$ spanned by $(1, 1, 0)$. Find vectors $v \in V$ and $w \in W$ so $v + w = (0, 0, 1)$.

[5] Let V be the subspace of $\mathbb{R}^3$ consisting of all solutions to the equation $x + y - z = 0$. Let W be the subspace of $\mathbb{R}^3$ spanned by $(1, 0, 4)$. Find vectors $v \in V$ and $w \in W$ so $v + w = (1, 1, 1)$.

[6] Let V be the subspace of $\mathbb{R}^3$ consisting of all solutions to the equation $x + 2y + z = 0$. Let W be the subspace of $\mathbb{R}^3$ spanned by $(1, 1, 1)$. Find vectors $v \in V$ and $w \in W$ so $v + w = (1, 1, 0)$.

(S11 Exam 1) (Solutions)

[1] What is the set of all solutions to the following system of equations?

$$\begin{bmatrix} 3 & 1 & 1 \\ 2 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ 5 \end{bmatrix}$$

[2] What is the set of all solutions to the following system of equations?

$$\begin{bmatrix} 0 & 1 & 1 & 5 & 0 & 9 \\ 0 & 0 & 1 & 3 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 6 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix}$$

[3] Use Gaussian elimination to find the inverse of the matrix

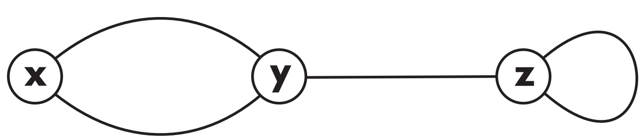
$$A = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 3 & 0 \end{bmatrix}$$

[4] Express A as a product of elementary matrices, where

$$A = \begin{bmatrix} 0 & 3 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

[5] Let $A : \mathbb{R}^2 \to \mathbb{R}^2$ be the matrix that flips the plane $\mathbb{R}^2$ across the line $3x = y$. Find A.

[6] Using matrix multiplication, count the number of paths of length four from y to itself.



(S11 Exam 2) (Solutions)

[1] Find a basis for the rowspace of the following matrix. Extend this basis to a basis for all of $\mathbb{R}^4$.

$$\begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$