3×3 Exercise Set M (recurrence relations)

Linear Algebra, Dave Bayer, November 27, 2016

[1] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\lambda = -1, 1, 2 \qquad A^n = \frac{(-1)^n}{6} \begin{bmatrix} 3 & -2 & -1 \\ -3 & 2 & 1 \\ -3 & 2 & 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} + \frac{2^n}{3} \begin{bmatrix} 0 & 1 & -1 \\ 0 & 2 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & -1 \\ -4 & 4 & 1 \end{bmatrix}$$

$$\lambda = -3, -1, 1 \qquad A^n = \frac{(-3)^n}{8} \begin{bmatrix} 0 & -4 & -1 \\ 0 & 12 & 3 \\ 0 & -16 & -4 \end{bmatrix} + \frac{(-1)^n}{4} \begin{bmatrix} 2 & 2 & 1 \\ -2 & -2 & -1 \\ 8 & 8 & 4 \end{bmatrix} + \frac{1}{8} \begin{bmatrix} 4 & 0 & -1 \\ 4 & 0 & -1 \\ -16 & 0 & 4 \end{bmatrix}$$

[3] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 4 & -3 \\ -4 & 2 & 2 \end{bmatrix}$$

$$\lambda = 1, 2, 3 \qquad A^n = \frac{1}{2} \begin{bmatrix} 9 & -1 & -3 \\ 9 & -1 & -3 \\ 18 & -2 & -6 \end{bmatrix} + 2^n \begin{bmatrix} -6 & 0 & 3 \\ -12 & 0 & 6 \\ -14 & 0 & 7 \end{bmatrix} + \frac{3^n}{2} \begin{bmatrix} 5 & 1 & -3 \\ 15 & 3 & -9 \\ 10 & 2 & -6 \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\lambda = -1, 1, 2 \qquad A^n = \frac{(-1)^n}{3} \begin{bmatrix} 0 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 1 \\ 2 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix} + \frac{2^n}{3} \begin{bmatrix} -3 & 1 & -2 \\ -6 & 2 & -4 \\ 6 & -2 & 4 \end{bmatrix}$$

[5] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 4 & -2 \\ -4 & 2 & 2 \end{bmatrix}$$

$$\lambda = 1, 2, 3 \qquad A^n \;=\; \frac{1}{2}\begin{bmatrix} 7 & -1 & -2 \\ 7 & -1 & -2 \\ 14 & -2 & -4 \end{bmatrix} + 2^n \begin{bmatrix} -4 & 0 & 2 \\ -8 & 0 & 4 \\ -10 & 0 & 5 \end{bmatrix} + \frac{3^n}{2}\begin{bmatrix} 3 & 1 & -2 \\ 9 & 3 & -6 \\ 6 & 2 & -4 \end{bmatrix}$$

[6] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & -4 \\ -1 & 2 & 2 \end{bmatrix}$$

$$\lambda = -2, -1, 1 \qquad A^n \;=\; \frac{(-2)^n}{3}\begin{bmatrix} 0 & -4 & -4 \\ 0 & 8 & 8 \\ 0 & -5 & -5 \end{bmatrix} + \frac{(-1)^n}{2}\begin{bmatrix} 1 & 3 & 4 \\ -1 & -3 & -4 \\ 1 & 3 & 4 \end{bmatrix} + \frac{1}{6}\begin{bmatrix} 3 & -1 & -4 \\ 3 & -1 & -4 \\ -3 & 1 & 4 \end{bmatrix}$$

[7] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} 0 & 1 & 0 \\ -2 & -1 & 1 \\ 4 & 4 & -1 \end{bmatrix}$$

$$\lambda = -2, -1, 1 \qquad A^n \;=\; \frac{(-2)^n}{3}\begin{bmatrix} -3 & -1 & 1 \\ 6 & 2 & -2 \\ -12 & -4 & 4 \end{bmatrix} + \frac{(-1)^n}{2}\begin{bmatrix} 4 & 0 & -1 \\ -4 & 0 & 1 \\ 8 & 0 & -2 \end{bmatrix} + \frac{1}{6}\begin{bmatrix} 0 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 8 & 4 \end{bmatrix}$$

[8] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ -2 & 2 & 1 \end{bmatrix}$$

$$\lambda = -1, 1, 3 \qquad A^n \;=\; \frac{(-1)^n}{8}\begin{bmatrix} 4 & -2 & 1 \\ -4 & 2 & -1 \\ 8 & -4 & 2 \end{bmatrix} + \frac{1}{4}\begin{bmatrix} 2 & 0 & -1 \\ 2 & 0 & -1 \\ -4 & 0 & 2 \end{bmatrix} + \frac{3^n}{8}\begin{bmatrix} 0 & 2 & 1 \\ 0 & 6 & 3 \\ 0 & 4 & 2 \end{bmatrix}$$

3×3 Exercise Set N (recurrence relations, repeated roots)

Linear Algebra, Dave Bayer, November 27, 2016

[1] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 4 & 1 & -2 \\ 3 & 1 & -1 \end{bmatrix}$$

$$\lambda = -2, 1, 1 \qquad A^n = \frac{(-2)^n}{9}\begin{bmatrix} 5 & -1 & -2 \\ -10 & 2 & 4 \\ -5 & 1 & 2 \end{bmatrix} + \frac{1}{9}\begin{bmatrix} 4 & 1 & 2 \\ 10 & 7 & -4 \\ 5 & -1 & 7 \end{bmatrix} + \frac{n}{3}\begin{bmatrix} 2 & 2 & -2 \\ 2 & 2 & -2 \\ 4 & 4 & -4 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\lambda = -2, 1, 1 \qquad A^n = \frac{(-2)^n}{9}\begin{bmatrix} 4 & -3 & -1 \\ -8 & 6 & 2 \\ 4 & -3 & -1 \end{bmatrix} + \frac{1}{9}\begin{bmatrix} 5 & 3 & 1 \\ 8 & 3 & -2 \\ -4 & 3 & 10 \end{bmatrix} + \frac{n}{3}\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

[3] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -2 & -2 & -1 \\ -2 & -1 & -2 \end{bmatrix}$$

$$\lambda = -2, -1, -1 \qquad A^n = (-2)^n\begin{bmatrix} -1 & 0 & -1 \\ 2 & 0 & 2 \\ 2 & 0 & 2 \end{bmatrix} + (-1)^n\begin{bmatrix} 2 & 0 & 1 \\ -2 & 1 & -2 \\ -2 & 0 & -1 \end{bmatrix} + n\,(-1)^{n-1}\begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 2 & -3 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\lambda = -1, 1, 1 \qquad A^n = \frac{(-1)^n}{4}\begin{bmatrix} 3 & 0 & -3 \\ -3 & 0 & 3 \\ -1 & 0 & 1 \end{bmatrix} + \frac{1}{4}\begin{bmatrix} 1 & 0 & 3 \\ 3 & 4 & -3 \\ 1 & 0 & 3 \end{bmatrix} + \frac{n}{2}\begin{bmatrix} 1 & 2 & -3 \\ 1 & 2 & -3 \\ 1 & 2 & -3 \end{bmatrix}$$

[5] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -2 & -4 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\lambda = 1, -2, -2 \qquad A^n \;=\; \frac{1}{9}\begin{bmatrix} 2 & 0 & 2 \\ 2 & 0 & 2 \\ 7 & 0 & 7 \end{bmatrix} + \frac{(-2)^n}{9}\begin{bmatrix} 7 & 0 & -2 \\ -2 & 9 & -2 \\ -7 & 0 & 2 \end{bmatrix} + \frac{n\,(-2)^{n-1}}{3}\begin{bmatrix} 4 & 3 & -2 \\ -8 & -6 & 4 \\ -4 & -3 & 2 \end{bmatrix}$$

[6] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} 0 & 1 & 0 \\ 2 & 3 & -4 \\ 1 & 2 & -3 \end{bmatrix}$$

$$\lambda = 2, -1, -1 \qquad A^n \;=\; \frac{2^n}{9}\begin{bmatrix} 3 & 5 & -4 \\ 6 & 10 & -8 \\ 3 & 5 & -4 \end{bmatrix} + \frac{(-1)^n}{9}\begin{bmatrix} 6 & -5 & 4 \\ -6 & -1 & 8 \\ -3 & -5 & 13 \end{bmatrix} + \frac{n\,(-1)^{n-1}}{3}\begin{bmatrix} 0 & -2 & 4 \\ 0 & 2 & -4 \\ 0 & 1 & -2 \end{bmatrix}$$

[7] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} 0 & 1 & 0 \\ 4 & -3 & 3 \\ -4 & -2 & -2 \end{bmatrix}$$

$$\lambda = -1, -2, -2 \qquad A^n \;=\; (-1)^n\begin{bmatrix} 8 & 1 & 3 \\ -8 & -1 & -3 \\ -16 & -2 & -6 \end{bmatrix} + (-2)^n\begin{bmatrix} -7 & -1 & -3 \\ 8 & 2 & 3 \\ 16 & 2 & 7 \end{bmatrix} + n\,(-2)^{n-1}\begin{bmatrix} -6 & 0 & -3 \\ 12 & 0 & 6 \\ 12 & 0 & 6 \end{bmatrix}$$

[8] Find A^n where A is the matrix

$$A \;=\; \begin{bmatrix} 0 & 1 & 0 \\ 4 & -1 & -1 \\ 4 & 2 & -2 \end{bmatrix}$$

$$\lambda = 1, -2, -2 \qquad A^n \;=\; \frac{1}{9}\begin{bmatrix} 8 & 3 & -1 \\ 8 & 3 & -1 \\ 16 & 6 & -2 \end{bmatrix} + \frac{(-2)^n}{9}\begin{bmatrix} 1 & -3 & 1 \\ -8 & 6 & 1 \\ -16 & -6 & 11 \end{bmatrix} + \frac{n\,(-2)^{n-1}}{3}\begin{bmatrix} -2 & 0 & 1 \\ 4 & 0 & -2 \\ -4 & 0 & 2 \end{bmatrix}$$

3×3 Exercise Set O (recurrence relations, identical roots)

Linear Algebra, Dave Bayer, November 27, 2016

[1] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 4 & 3 \\ 4 & -2 & 2 \end{bmatrix}$$

$$\lambda = 2, 2, 2$$

$$A^n = 2^n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + n\,2^{n-1} \begin{bmatrix} -2 & 1 & 0 \\ 2 & 2 & 3 \\ 4 & -2 & 0 \end{bmatrix} + \frac{n(n-1)\,2^{n-2}}{2} \begin{bmatrix} 6 & 0 & 3 \\ 12 & 0 & 6 \\ -12 & 0 & -6 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 2 & -1 \\ -3 & 3 & 1 \end{bmatrix}$$

$$\lambda = 1, 1, 1$$

$$A^n = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + n \begin{bmatrix} -1 & 1 & 0 \\ 2 & 1 & -1 \\ -3 & 3 & 0 \end{bmatrix} + \frac{n(n-1)}{2} \begin{bmatrix} 3 & 0 & -1 \\ 3 & 0 & -1 \\ 9 & 0 & -3 \end{bmatrix}$$

[3] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 2 \\ -1 & -1 & -1 \end{bmatrix}$$

$$\lambda = -1, -1, -1$$

$$A^n = (-1)^n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + n\,(-1)^{n-1} \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 2 \\ -1 & -1 & 0 \end{bmatrix} + \frac{n(n-1)\,(-1)^{n-2}}{2} \begin{bmatrix} 2 & 0 & 2 \\ -2 & 0 & -2 \\ -2 & 0 & -2 \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 2 & -1 \\ -4 & 4 & 1 \end{bmatrix}$$

$$\lambda = 1, 1, 1$$

$$A^n = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + n \begin{bmatrix} -1 & 1 & 0 \\ 3 & 1 & -1 \\ -4 & 4 & 0 \end{bmatrix} + \frac{n(n-1)}{2} \begin{bmatrix} 4 & 0 & -1 \\ 4 & 0 & -1 \\ 16 & 0 & -4 \end{bmatrix}$$

[5] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -4 & 2 & 1 \\ -3 & 3 & 1 \end{bmatrix}$$

$$\lambda = 1, 1, 1$$

$$A^n = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + n \begin{bmatrix} -1 & 1 & 0 \\ -4 & 1 & 1 \\ -3 & 3 & 0 \end{bmatrix} + \frac{n(n-1)}{2} \begin{bmatrix} -3 & 0 & 1 \\ -3 & 0 & 1 \\ -9 & 0 & 3 \end{bmatrix}$$

[6] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & -2 & -1 \\ 4 & 4 & -1 \end{bmatrix}$$

$$\lambda = -1, -1, -1$$

$$A^n = (-1)^n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + n\,(-1)^{n-1} \begin{bmatrix} 1 & 1 & 0 \\ 3 & -1 & -1 \\ 4 & 4 & 0 \end{bmatrix} + \frac{n(n-1)\,(-1)^{n-2}}{2} \begin{bmatrix} 4 & 0 & -1 \\ -4 & 0 & 1 \\ 16 & 0 & -4 \end{bmatrix}$$

[7] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & -2 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\lambda = -1, -1, -1$$

$$A^n = (-1)^n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + n\,(-1)^{n-1} \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & -2 \\ 1 & 1 & 0 \end{bmatrix} + \frac{n(n-1)\,(-1)^{n-2}}{2} \begin{bmatrix} 2 & 0 & -2 \\ -2 & 0 & 2 \\ 2 & 0 & -2 \end{bmatrix}$$

[8] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\lambda = 1, 1, 1$$

$$A^n = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + n \begin{bmatrix} -1 & 1 & 0 \\ 1 & 1 & 1 \\ 2 & -2 & 0 \end{bmatrix} + \frac{n(n-1)}{2} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 0 & 1 \\ -4 & 0 & -2 \end{bmatrix}$$

3×3 Exercise Set P (differential equations)

Linear Algebra, Dave Bayer, November 27, 2016

[1] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & -1 & 1 \\ -1 & -1 & -1 \end{bmatrix}$$

$$\lambda = -2, -1, 1 \qquad e^{At} = \frac{e^{-2t}}{3} \begin{bmatrix} 2 & -1 & 1 \\ -4 & 2 & -2 \\ -2 & 1 & -1 \end{bmatrix} + \frac{e^{-t}}{2} \begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 3 & 0 & 3 \end{bmatrix} + \frac{e^{t}}{6} \begin{bmatrix} 5 & 2 & 1 \\ 5 & 2 & 1 \\ -5 & -2 & -1 \end{bmatrix}$$

[2] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 1 & 3 \\ -3 & -2 & -4 \end{bmatrix}$$

$$\lambda = -3, -1, 1 \qquad e^{At} = \frac{e^{-3t}}{8} \begin{bmatrix} 2 & 1 & 3 \\ -6 & -3 & -9 \\ 6 & 3 & 9 \end{bmatrix} + \frac{e^{-t}}{4} \begin{bmatrix} 0 & -3 & -3 \\ 0 & 3 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \frac{e^{t}}{8} \begin{bmatrix} 6 & 5 & 3 \\ 6 & 5 & 3 \\ -6 & -5 & -3 \end{bmatrix}$$

[3] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -2 & 4 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\lambda = -3, -1, 1 \qquad e^{At} = \frac{e^{-3t}}{4} \begin{bmatrix} -1 & -1 & 2 \\ 3 & 3 & -6 \\ -1 & -1 & 2 \end{bmatrix} + e^{-t} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + \frac{e^{t}}{4} \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -4 & -4 & 1 \\ 4 & 2 & -3 \end{bmatrix}$$

$$\lambda = -4, -2, -1 \qquad e^{At} = \frac{e^{-4t}}{6} \begin{bmatrix} -2 & -1 & 1 \\ 8 & 4 & -4 \\ -8 & -4 & 4 \end{bmatrix} + \frac{e^{-2t}}{2} \begin{bmatrix} 0 & -1 & -1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix} + \frac{e^{-t}}{3} \begin{bmatrix} 4 & 2 & 1 \\ -4 & -2 & -1 \\ 4 & 2 & 1 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -4 & 4 & -3 \\ -2 & 1 & -2 \end{bmatrix}$$

$$\lambda = -1, 1, 2 \qquad e^{At} \;=\; \frac{e^{-t}}{6}\begin{bmatrix} -2 & 1 & -3 \\ 2 & -1 & 3 \\ 6 & -3 & 9 \end{bmatrix} + \frac{e^{t}}{2}\begin{bmatrix} 6 & -3 & 3 \\ 6 & -3 & 3 \\ -2 & 1 & -1 \end{bmatrix} + \frac{e^{2t}}{3}\begin{bmatrix} -5 & 4 & -3 \\ -10 & 8 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A \;=\; \begin{bmatrix} 0 & 1 & 0 \\ -1 & 1 & -1 \\ 3 & -3 & 1 \end{bmatrix}$$

$$\lambda = -1, 1, 2 \qquad e^{At} \;=\; \frac{e^{-t}}{6}\begin{bmatrix} 1 & -2 & -1 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix} + \frac{e^{t}}{2}\begin{bmatrix} 3 & 0 & 1 \\ 3 & 0 & 1 \\ -3 & 0 & -1 \end{bmatrix} + \frac{e^{2t}}{3}\begin{bmatrix} -2 & 1 & -1 \\ -4 & 2 & -2 \\ 6 & -3 & 3 \end{bmatrix}$$

[7] Find e^{At} where A is the matrix

$$A \;=\; \begin{bmatrix} 0 & 1 & 0 \\ 1 & -3 & -3 \\ 3 & 1 & -3 \end{bmatrix}$$

$$\lambda = -3, -2, -1 \qquad e^{At} \;=\; \frac{e^{-3t}}{2}\begin{bmatrix} 3 & 0 & -3 \\ -9 & 0 & 9 \\ 1 & 0 & -1 \end{bmatrix} + e^{-2t}\begin{bmatrix} -4 & -1 & 3 \\ 8 & 2 & -6 \\ -4 & -1 & 3 \end{bmatrix} + \frac{e^{-t}}{2}\begin{bmatrix} 7 & 2 & -3 \\ -7 & -2 & 3 \\ 7 & 2 & -3 \end{bmatrix}$$

[8] Find e^{At} where A is the matrix

$$A \;=\; \begin{bmatrix} 0 & 1 & 0 \\ 3 & 4 & 3 \\ 4 & -2 & 2 \end{bmatrix}$$

$$\lambda = 1, 2, 3 \qquad e^{At} \;=\; \frac{e^{t}}{2}\begin{bmatrix} 9 & -1 & 3 \\ 9 & -1 & 3 \\ -18 & 2 & -6 \end{bmatrix} + e^{2t}\begin{bmatrix} -6 & 0 & -3 \\ -12 & 0 & -6 \\ 14 & 0 & 7 \end{bmatrix} + \frac{e^{3t}}{2}\begin{bmatrix} 5 & 1 & 3 \\ 15 & 3 & 9 \\ -10 & -2 & -6 \end{bmatrix}$$

3×3 Exercise Set Q (differential equations, repeated roots)

Linear Algebra, Dave Bayer, November 27, 2016

[1] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -4 & -3 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\lambda = 1, -2, -2 \qquad e^{At} = \frac{e^{t}}{3}\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ -2 & 0 & 2 \end{bmatrix} + \frac{e^{-2t}}{3}\begin{bmatrix} 2 & 0 & 1 \\ -1 & 3 & 1 \\ 2 & 0 & 1 \end{bmatrix} + te^{-2t}\begin{bmatrix} 1 & 1 & 1 \\ -2 & -2 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

[2] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -4 & -2 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\lambda = -3, -1, -1 \qquad e^{At} = \frac{e^{-3t}}{2}\begin{bmatrix} 0 & -1 & -1 \\ 0 & 3 & 3 \\ 0 & -1 & -1 \end{bmatrix} + \frac{e^{-t}}{2}\begin{bmatrix} 2 & 1 & 1 \\ 0 & -1 & -3 \\ 0 & 1 & 3 \end{bmatrix} + te^{-t}\begin{bmatrix} 1 & 0 & -1 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

[3] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -3 & 2 & 1 \\ 4 & 2 & -2 \end{bmatrix}$$

$$\lambda = -2, 1, 1 \qquad e^{At} = \frac{e^{-2t}}{9}\begin{bmatrix} -2 & 0 & 1 \\ 4 & 0 & -2 \\ -22 & 0 & 11 \end{bmatrix} + \frac{e^{t}}{9}\begin{bmatrix} 11 & 0 & -1 \\ -4 & 9 & 2 \\ 22 & 0 & -2 \end{bmatrix} + \frac{te^{t}}{3}\begin{bmatrix} -5 & 3 & 1 \\ -5 & 3 & 1 \\ -10 & 6 & 2 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 4 & -1 & 2 \\ 3 & -1 & 1 \end{bmatrix}$$

$$\lambda = 2, -1, -1 \qquad e^{At} = \frac{e^{2t}}{9}\begin{bmatrix} 5 & 1 & 2 \\ 10 & 2 & 4 \\ 5 & 1 & 2 \end{bmatrix} + \frac{e^{-t}}{9}\begin{bmatrix} 4 & -1 & -2 \\ -10 & 7 & -4 \\ -5 & -1 & 7 \end{bmatrix} + \frac{te^{-t}}{3}\begin{bmatrix} -2 & 2 & -2 \\ 2 & -2 & 2 \\ 4 & -4 & 4 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -4 & -3 & -2 \\ -4 & -2 & -3 \end{bmatrix}$$

$$\lambda = -4, -1, -1 \qquad e^{At} = \frac{e^{-4t}}{9}\begin{bmatrix} -3 & -1 & -2 \\ 12 & 4 & 8 \\ 12 & 4 & 8 \end{bmatrix} + \frac{e^{-t}}{9}\begin{bmatrix} 12 & 1 & 2 \\ -12 & 5 & -8 \\ -12 & -4 & 1 \end{bmatrix} + \frac{te^{-t}}{3}\begin{bmatrix} 0 & 2 & -2 \\ 0 & -2 & 2 \\ 0 & -2 & 2 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -4 & -1 & -3 \\ -4 & -2 & -2 \end{bmatrix}$$

$$\lambda = 1, -2, -2 \qquad e^{At} = \frac{e^{t}}{3}\begin{bmatrix} 0 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & -2 & 2 \end{bmatrix} + \frac{e^{-2t}}{3}\begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix} + te^{-2t}\begin{bmatrix} 2 & 0 & 1 \\ -4 & 0 & -2 \\ -4 & 0 & -2 \end{bmatrix}$$

[7] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -1 & -1 \\ -1 & -3 & 1 \end{bmatrix}$$

$$\lambda = 2, -1, -1 \qquad e^{At} = \frac{e^{2t}}{9}\begin{bmatrix} 0 & 1 & -1 \\ 0 & 2 & -2 \\ 0 & -7 & 7 \end{bmatrix} + \frac{e^{-t}}{9}\begin{bmatrix} 9 & -1 & 1 \\ 0 & 7 & 2 \\ 0 & 7 & 2 \end{bmatrix} + \frac{te^{-t}}{3}\begin{bmatrix} 3 & 2 & 1 \\ -3 & -2 & -1 \\ -3 & -2 & -1 \end{bmatrix}$$

[8] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 4 & 2 \\ 3 & -1 & 3 \end{bmatrix}$$

$$\lambda = 1, 3, 3 \qquad e^{At} = \frac{e^{t}}{2}\begin{bmatrix} 4 & -1 & 1 \\ 4 & -1 & 1 \\ -4 & 1 & -1 \end{bmatrix} + \frac{e^{3t}}{2}\begin{bmatrix} -2 & 1 & -1 \\ -4 & 3 & -1 \\ 4 & -1 & 3 \end{bmatrix} + te^{3t}\begin{bmatrix} 1 & 0 & 1 \\ 3 & 0 & 3 \\ -1 & 0 & -1 \end{bmatrix}$$

3×3 Exercise Set R (differential equations, identical roots)

Linear Algebra, Dave Bayer, November 27, 2016

[1] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -4 & 2 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\lambda = 1, 1, 1$$

$$e^{At} = e^t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^t \begin{bmatrix} -1 & 1 & 0 \\ -4 & 1 & 3 \\ -1 & 1 & 0 \end{bmatrix} + \frac{t^2e^t}{2} \begin{bmatrix} -3 & 0 & 3 \\ -3 & 0 & 3 \\ -3 & 0 & 3 \end{bmatrix}$$

[2] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -3 & 4 & 1 \\ 2 & -1 & 2 \end{bmatrix}$$

$$\lambda = 2, 2, 2$$

$$e^{At} = e^{2t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^{2t} \begin{bmatrix} -2 & 1 & 0 \\ -3 & 2 & 1 \\ 2 & -1 & 0 \end{bmatrix} + \frac{t^2e^{2t}}{2} \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \\ -1 & 0 & -1 \end{bmatrix}$$

[3] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 2 & -1 \\ -3 & 3 & 1 \end{bmatrix}$$

$$\lambda = 1, 1, 1$$

$$e^{At} = e^t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^t \begin{bmatrix} -1 & 1 & 0 \\ 2 & 1 & -1 \\ -3 & 3 & 0 \end{bmatrix} + \frac{t^2e^t}{2} \begin{bmatrix} 3 & 0 & -1 \\ 3 & 0 & -1 \\ 9 & 0 & -3 \end{bmatrix}$$

[4] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -3 & 2 & 1 \\ -4 & -1 & 4 \end{bmatrix}$$

$$\lambda = 2, 2, 2$$

$$e^{At} = e^{2t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^{2t} \begin{bmatrix} -2 & 1 & 0 \\ -3 & 0 & 1 \\ -4 & -1 & 2 \end{bmatrix} + \frac{t^2 e^{2t}}{2} \begin{bmatrix} 1 & -2 & 1 \\ 2 & -4 & 2 \\ 3 & -6 & 3 \end{bmatrix}$$

[5] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 2 & 1 \\ 4 & -3 & 4 \end{bmatrix}$$

$$\lambda = 2, 2, 2$$

$$e^{At} = e^{2t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^{2t} \begin{bmatrix} -2 & 1 & 0 \\ -1 & 0 & 1 \\ 4 & -3 & 2 \end{bmatrix} + \frac{t^2 e^{2t}}{2} \begin{bmatrix} 3 & -2 & 1 \\ 6 & -4 & 2 \\ 3 & -2 & 1 \end{bmatrix}$$

[6] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 2 & -1 \\ -4 & 3 & 4 \end{bmatrix}$$

$$\lambda = 2, 2, 2$$

$$e^{At} = e^{2t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^{2t} \begin{bmatrix} -2 & 1 & 0 \\ -1 & 0 & -1 \\ -4 & 3 & 2 \end{bmatrix} + \frac{t^2 e^{2t}}{2} \begin{bmatrix} 3 & -2 & -1 \\ 6 & -4 & -2 \\ -3 & 2 & 1 \end{bmatrix}$$

[7] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -3 & 2 & -1 \\ 4 & 1 & 4 \end{bmatrix}$$

$$\lambda = 2, 2, 2$$

$$e^{At} = e^{2t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^{2t} \begin{bmatrix} -2 & 1 & 0 \\ -3 & 0 & -1 \\ 4 & 1 & 2 \end{bmatrix} + \frac{t^2 e^{2t}}{2} \begin{bmatrix} 1 & -2 & -1 \\ 2 & -4 & -2 \\ -3 & 6 & 3 \end{bmatrix}$$

[8] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & -2 & -2 \\ 2 & 2 & -1 \end{bmatrix}$$

$$\lambda = -1, -1, -1$$

$$e^{At} = e^{-t}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + te^{-t}\begin{bmatrix} 1 & 1 & 0 \\ 3 & -1 & -2 \\ 2 & 2 & 0 \end{bmatrix} + \frac{t^2 e^{-t}}{2}\begin{bmatrix} 4 & 0 & -2 \\ -4 & 0 & 2 \\ 8 & 0 & -4 \end{bmatrix}$$

3×3 Exercise Set S (Markov chains)

Linear Algebra, Dave Bayer, November 27, 2016

[1] Find A^n where A is the matrix

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\lambda = 2, 3, 5 \qquad A^n = \frac{2^n}{3}\begin{bmatrix} 0 & 0 & 0 \\ -1 & 2 & -1 \\ 1 & -2 & 1 \end{bmatrix} + \frac{3^n}{2}\begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix} + \frac{5^n}{6}\begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & 0 \\ 2 & 1 & 2 \end{bmatrix}$$

$$\lambda = 0, 2, 4 \qquad A^n = \frac{0^n}{2}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} + \frac{2^n}{2}\begin{bmatrix} 0 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix} + \frac{4^n}{2}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

[3] Find A^n where A is the matrix

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 0 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\lambda = 1, 3, 5 \qquad A^n = \frac{1}{4}\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & -3 \\ -1 & -1 & 3 \end{bmatrix} + \frac{3^n}{2}\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} + \frac{5^n}{4}\begin{bmatrix} 2 & 2 & 2 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 3 & 0 \\ 0 & 1 & 4 \end{bmatrix}$$

$$\lambda = 1, 3, 5 \qquad A^n = \frac{1}{8}\begin{bmatrix} 6 & -2 & -2 \\ -9 & 3 & 3 \\ 3 & -1 & -1 \end{bmatrix} + \frac{3^n}{4}\begin{bmatrix} 0 & 0 & 0 \\ 3 & 1 & -3 \\ -3 & -1 & 3 \end{bmatrix} + \frac{5^n}{8}\begin{bmatrix} 2 & 2 & 2 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$$

[5] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix}$$

$$\lambda = 1, 3, 5 \qquad A^n = \frac{1}{8}\begin{bmatrix} 5 & -3 & 1 \\ -5 & 3 & -1 \\ 0 & 0 & 0 \end{bmatrix} + \frac{3^n}{4}\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -2 & -2 & 2 \end{bmatrix} + \frac{5^n}{8}\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix}$$

[6] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\lambda = 0, 2, 4 \qquad A^n = \frac{0^n}{8}\begin{bmatrix} 3 & -9 & 3 \\ -2 & 6 & -2 \\ -1 & 3 & -1 \end{bmatrix} + \frac{2^n}{4}\begin{bmatrix} 1 & 3 & -3 \\ 0 & 0 & 0 \\ -1 & -3 & 3 \end{bmatrix} + \frac{4^n}{8}\begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

[7] Find A^n where A is the matrix

$$A = \begin{bmatrix} 4 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 3 & 3 \end{bmatrix}$$

$$\lambda = 1, 3, 5 \qquad A^n = \frac{1}{8}\begin{bmatrix} -1 & 3 & -1 \\ -2 & 6 & -2 \\ 3 & -9 & 3 \end{bmatrix} + \frac{3^n}{4}\begin{bmatrix} 3 & -3 & -1 \\ 0 & 0 & 0 \\ -3 & 3 & 1 \end{bmatrix} + \frac{5^n}{8}\begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

[8] Find A^n where A is the matrix

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 4 \end{bmatrix}$$

$$\lambda = 1, 3, 5 \qquad A^n = \frac{1}{8}\begin{bmatrix} 3 & -5 & -1 \\ -3 & 5 & 1 \\ 0 & 0 & 0 \end{bmatrix} + \frac{3^n}{4}\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -2 & -2 & 2 \end{bmatrix} + \frac{5^n}{8}\begin{bmatrix} 3 & 3 & 3 \\ 1 & 1 & 1 \\ 4 & 4 & 4 \end{bmatrix}$$