### $3 \times 3$ Exercise Set M (recurrence relations)

Linear Algebra, Dave Bayer, November 27, 2016

[1] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

[2] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & -1 \\ -4 & 4 & 1 \end{bmatrix}$$

[3] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 4 & -3 \\ -4 & 2 & 2 \end{bmatrix}$$

[4] Find  $A^n$  where A is the matrix

	0	1	0 ]
A =	-2	1	-2
	2	-2	$\begin{bmatrix} 0\\ -2\\ 1 \end{bmatrix}$

[5] Find  $A^n$  where A is the matrix

	0	1	0
A =	1	4	-2
	4	2	$\begin{bmatrix} 0\\ -2\\ 2 \end{bmatrix}$

[6] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & -4 \\ -1 & 2 & 2 \end{bmatrix}$$

[7] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -2 & -1 & 1 \\ 4 & 4 & -1 \end{bmatrix}$$

[8] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ -2 & 2 & 1 \end{bmatrix}$$

### $3 \times 3$ Exercise Set N (recurrence relations, repeated roots)

Linear Algebra, Dave Bayer, November 27, 2016

[1] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 4 & 1 & -2 \\ 3 & 1 & -1 \end{bmatrix}$$

[2] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

[3] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -2 & -2 & -1 \\ -2 & -1 & -2 \end{bmatrix}$$

[4] Find  $A^n$  where A is the matrix

	0	1	0 ]
A =		2	
	1	1	-1

[5] Find  $A^n$  where A is the matrix

	0	1	0	
A =	-2	-4	2	
A =	1	-1	1	

[6] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 3 & -4 \\ 1 & 2 & -3 \end{bmatrix}$$

[7] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 4 & -3 & 3 \\ -4 & -2 & -2 \end{bmatrix}$$

[8] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 4 & -1 & -1 \\ 4 & 2 & -2 \end{bmatrix}$$

## $3 \times 3$ Exercise Set O (recurrence relations, identical roots)

Linear Algebra, Dave Bayer, November 27, 2016

[1] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 4 & 3 \\ 4 & -2 & 2 \end{bmatrix}$$

[2] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 2 & -1 \\ -3 & 3 & 1 \end{bmatrix}$$

[3] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 2 \\ -1 & -1 & -1 \end{bmatrix}$$

[4] Find  $A^n$  where A is the matrix

	0	1	0 ]
A =	3	2	-1
	4	4	$\begin{bmatrix} -1\\ 1 \end{bmatrix}$

[5] Find  $A^n$  where A is the matrix

	0	1	0	
A =	-4			
	3	3	1	

[6] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & -2 & -1 \\ 4 & 4 & -1 \end{bmatrix}$$

[7] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & -2 \\ 1 & 1 & -1 \end{bmatrix}$$

[8] Find  $A^n$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

# $3 \times 3$ Exercise Set P (differential equations)

Linear Algebra, Dave Bayer, November 27, 2016

[1] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & -1 & 1 \\ -1 & -1 & -1 \end{bmatrix}$$

[2] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 1 & 3 \\ -3 & -2 & -4 \end{bmatrix}$$

[3] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -2 & 4 \\ 1 & 1 & -1 \end{bmatrix}$$

[4] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -4 & -4 & 1 \\ 4 & 2 & -3 \end{bmatrix}$$

[5] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -4 & 4 & -3 \\ -2 & 1 & -2 \end{bmatrix}$$

[6] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 1 & -1 \\ 3 & -3 & 1 \end{bmatrix}$$

[7] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -3 & -3 \\ 3 & 1 & -3 \end{bmatrix}$$

[8] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 4 & 3 \\ 4 & -2 & 2 \end{bmatrix}$$

# $3 \times 3$ Exercise Set Q (differential equations, repeated roots)

Linear Algebra, Dave Bayer, November 27, 2016

[1] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -4 & -3 \\ -1 & 1 & 1 \end{bmatrix}$$

[2] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -4 & -2 \\ 1 & 1 & -1 \end{bmatrix}$$

[3] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -3 & 2 & 1 \\ 4 & 2 & -2 \end{bmatrix}$$

[4] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 4 & -1 & 2 \\ 3 & -1 & 1 \end{bmatrix}$$

[5] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -4 & -3 & -2 \\ -4 & -2 & -3 \end{bmatrix}$$

[6] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -4 & -1 & -3 \\ -4 & -2 & -2 \end{bmatrix}$$

[7] Find  $e^{At}$  where A is the matrix

$$A \;=\; \left[ \begin{array}{rrrr} 0 & 1 & 0 \\ -1 & -1 & -1 \\ -1 & -3 & 1 \end{array} \right]$$

[8] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 4 & 2 \\ 3 & -1 & 3 \end{bmatrix}$$

# $3 \times 3$ Exercise Set R (differential equations, identical roots)

Linear Algebra, Dave Bayer, November 27, 2016

[1] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -4 & 2 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

[2] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -3 & 4 & 1 \\ 2 & -1 & 2 \end{bmatrix}$$

[3] Find  $e^{At}$  where A is the matrix

	0	1	0 ]
A =	2	2	-1
	3	3	1

[4] Find  $e^{At}$  where A is the matrix

	0	1	0	
A =	-3	2	1	
	$\begin{bmatrix} 0\\ -3\\ -4 \end{bmatrix}$	-1	4	

[5] Find  $e^{At}$  where A is the matrix

0	1	0	1
-1	2	1	
4	-3	4_	
	$\left[ egin{array}{c} 0 \ -1 \ 4 \end{array}  ight]$	$\begin{bmatrix} 0 & 1 \\ -1 & 2 \\ 4 & -3 \end{bmatrix}$	-1 2 1

[6] Find  $e^{At}$  where A is the matrix

	0	1	0 ]
A =	-1	2	-1
A =	4	3	4

[7] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -3 & 2 & -1 \\ 4 & 1 & 4 \end{bmatrix}$$

[8] Find  $e^{At}$  where A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & -2 & -2 \\ 2 & 2 & -1 \end{bmatrix}$$

## $3 \times 3$ Exercise Set S (Markov chains)

Linear Algebra, Dave Bayer, November 27, 2016

[1] Find  $A^n$  where A is the matrix

[1] Find A <sup>n</sup> where A is the matrix	$A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{bmatrix}$
[2] Find A <sup>n</sup> where A is the matrix	$A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & 0 \\ 2 & 1 & 2 \end{bmatrix}$
[3] Find A <sup>n</sup> where A is the matrix	$A = \begin{bmatrix} 4 & 1 & 1 \\ 0 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$
[4] Find A <sup>n</sup> where A is the matrix	$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 3 & 0 \\ 0 & 1 & 4 \end{bmatrix}$
[5] Find A <sup>n</sup> where A is the matrix	$A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix}$
[6] Find A <sup>n</sup> where A is the matrix	$A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 3 \end{bmatrix}$
[7] Find A <sup>n</sup> where A is the matrix	$A = \begin{bmatrix} 4 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 3 & 3 \end{bmatrix}$
[8] Find A <sup>n</sup> where A is the matrix	$A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 4 \end{bmatrix}$