F16 Study Guide for Exam 2

Linear Algebra, Dave Bayer

Our second exam will be held in class on Tuesday, November 15, 2016.

Exam 2 will consist of five questions. The following are the skills that one needs to learn for this exam:

- Compute the determinant of an $n \times n$ matrix, using any or a combination of the methods we have learned in class: Directly using an understanding of the formula as permutations, expansion by minors, tracking the effect of Gaussian elimination.
- Compute the inverse of an $n \times n$ matrix, using the formula. Note that we have studied streamlined ways to carry out this computation for 2×2 and 3×3 matrices.
- Compute a 3×3 matrix A from a description of the linear map, using change of coordinates and the formula for the 3×3 inverse.
- Using Cramer's rule, solve for one value or a ratio of values in a system of equations.
- Find the characteristic equation and a system of eigenvalues and eigenvectors for a 2×2 matrix. (The eigenvalues will be distinct integers.)
- Find a formula for the nth power of a 2×2 matrix. (The eigenvalues will be distinct integers.)
- Express a recurrence relation as a matrix, and solve for a specific value by taking a matrix power.
- Find a recurrence relation describing a sequence of determinants, and solve for a specific value.

This material is covered in chapters six and seven of Bretscher. You are encouraged to read these chapters carefully.

Homework will count as 10% of your course grade. The homework for Exam 2 is given below. You may turn in problems in batchs as you complete them; homework that is received early will receive more careful feedback. All homework must be submitted on or before the day of our exam. There is a homework box on the fourth floor of the Mathematics building for your section of Linear Algebra. Please turn in homework to the box corresponding to your section. Please write your uni very clearly on each page of homework.

Please hand in the following problems; they are the same in both the 5e and 4e editions of Bretscher. (You are encouraged to work similar problems for your own practice.)

- 6.1 [30], 6.2 [10], 6.3 [14]
- 7.1 [10], 7.2 [14], 7.3 [22], 7.4 [42], 7.5 [14]

What follows on the remaining pages of this study guide are practice problems for our second exam, taken from past semesters of the course. Note that some 3×3 matrix problems also appeared on our previous study guide. These problems should now be easier.

The sources for the following problems, along with many solutions, can be found on our Linear Algebra Course Materials page:

https://www.math.columbia.edu/~bayer/LinearAlgebra/

(F15 Homework 1) (Solutions)

[4] Find the matrix A such that

$$A \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 4 & 4 \\ 3 & 3 & 1 \end{bmatrix}$$

(F15 Homework 2)

[2] Find the 3×3 matrix that vanishes on the plane $3x - 2y + z = 0$, and maps the vector $(1, 0, 0)$ to itself.

[3] Find the 3×3 matrix that vanishes on the vector $(1, 0, 2)$, and stretches each vector in the plane $x + y = 0$ by a factor of 3.

[4] Find the 3×3 matrix that projects orthogonally onto the line

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} t$$

[5] Find the 3×3 matrix that projects orthogonally onto the plane

$$x - 2y + z = 0$$

(F15 Exam 2) (Solutions)

[2] Find the 3×3 matrix A such that

$$A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad A \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \qquad A \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

(F15 Homework 3)

[1] Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 1 & 3 & 3 & 1 \\ 1 & 4 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

[2] Find the determinant of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 & 1 & 1 \\ 1 & 3 & 1 & 2 & 1 \\ 5 & 1 & 4 & 1 & 1 \\ 1 & 2 & 1 & 3 & 1 \\ 3 & 3 & 3 & 3 & 3 \end{bmatrix}$$

[3] Find the inverse of the matrix

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

[4] Using Cramer's rule, solve for x in the system of equations

$$\begin{bmatrix} 3 & a & 2 \\ 2 & b & 3 \\ 1 & c & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

[5] Find the characteristic equation and a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 1 & 4 \\ 1 & -2 \end{bmatrix}$$

[7] Express $f(n)$ using a matrix power, and find $f(8)$, where

$$f(0) = -1, \quad f(1) = 2$$
$$f(n) = f(n-1) + f(n-2)$$

[8] Express $f(n)$ using a matrix power, and find $f(8)$, where

$$f(0) = 1, \quad f(1) = 1, \quad g(1) = 1$$
$$f(n) = f(n-1) + g(n-1)$$
$$g(n) = f(n-1) + f(n-2)$$

[9] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[\,] \qquad [\,1\,] \qquad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Find $f(0)$ and $f(1)$. Find a recurrence relation for $f(n)$. Express $f(n)$ using a matrix power. Find $f(8)$.

(F15 Exam 3)

[1] Find the determinant of the matrix

$$A = \begin{bmatrix} 3 & 6 & 6 & 1 & 1 \\ 1 & 3 & 6 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 1 & 1 & 6 \\ 1 & 1 & 1 & 1 & 3 \end{bmatrix}$$

[2] Using Cramer's rule, solve for z in the system of equations

$$\begin{bmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

[3] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[\,0\,] \qquad \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 2 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 2 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Find $f(10)$.

[4] Find a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 4 & 6 \\ 1 & 5 \end{bmatrix}$$

(F15 Homework 4)

[1] Find A^n where A is the matrix

$$A = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}$$

(F15 Final) (Solutions)

[2] Find the 3×3 matrix A such that

$$A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad A \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad A \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

[3] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$\begin{bmatrix} 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Find $f(8)$.

(F14 Practice 1) (Solutions)

[4] Find the matrix A such that

$$A \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ -3 & 3 & 0 \\ 0 & -3 & 0 \end{bmatrix}$$

(F14 Homework 1) (Solutions)

[4] Find the matrix A such that

$$A \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 3 \\ 0 & 0 & 3 \\ 0 & 0 & 3 \end{bmatrix}$$

(F14 8:40 Exam 1) (Solutions)

[4] Find the matrix A such that

$$A \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

(F14 11:40 Exam 1) (Solutions)

[4] Find the matrix A such that

$$A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix}$$

(F14 Practice 2) (Solutions)

[2] Find the 3×3 matrix that vanishes on the plane $x + y + z = 0$, and maps the vector $(1, 0, 0)$ to itself.

[3] Find the 3×3 matrix that vanishes on the vector $(1, 1, 1)$, and maps each point on the plane $x + y = 0$ to itself.

[4] Find the 3×3 matrix that projects orthogonally onto the line

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} t$$

[5] Find the 3×3 matrix that projects orthogonally onto the plane

$$x + y + z = 0$$

(F14 Homework 2) (Solutions)

[2] Find the 3×3 matrix that vanishes on the plane $4x + 2y + z = 0$, and maps the vector $(1, 1, 1)$ to itself.

[3] Find the 3×3 matrix that vanishes on the vector $(1, 1, 0)$, and maps each point on the plane $x + 2y + 2z = 0$ to itself.

[4] Find the 3×3 matrix that projects orthogonally onto the line

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} t$$

[5] Find the 3×3 matrix that projects orthogonally onto the plane

$$x + 2y + 3z = 0$$

(F14 8:40 Exam 2) (Solutions)

[1] Find the 3×3 matrix that maps the vector $(0, 1, 1)$ to $(0, 2, 2)$, and maps each point on the plane $x + y = 0$ to the zero vector.

[3] Find the 3×3 matrix that projects orthogonally onto the plane

$$x + 2y = 0$$

(F14 11:40 Exam 2) (Solutions)

[1] Find the 3×3 matrix that maps the vector $(1, 1, 1)$ to $(2, 2, 2)$, and maps each point on the plane $x+y+z = 0$ to itself.

[3] Find the 3×3 matrix that projects orthogonally onto the plane

$$x + y - 2z = 0$$

(F14 Practice 3) (Solutions)

[1] Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 3 & 4 & 1 & 5 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

[2] Find the determinant of the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 0 \\ 1 & 1 & 3 & 1 & 2 \\ 1 & 1 & 1 & 3 & 0 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix}$$

[3] Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

[4] Using Cramer's rule, solve for x in the system of equations

$$\begin{bmatrix} 3 & 1 & a \\ 2 & 1 & b \\ 1 & 1 & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 2 \end{bmatrix}$$

[5] Find the characteristic equation and a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

[7] Express $f(n)$ using a matrix power, and find $f(8)$, where

$$\begin{gathered} f(0) = 1, \quad f(1) = 2 \\ f(n) \;=\; 2\,f(n-1) \;-\; f(n-2) \end{gathered}$$

[8] Express $f(n)$ using a matrix power, and find $f(8)$, where

$$\begin{aligned} & f(0) = 1, \quad f(1) = 1, \quad g(1) = 2 \\ f(n) \;&=\; f(n-1) \;+\; g(n-1) \;+\; f(n-2) \\ g(n) \;&=\; f(n-1) \;-\; g(n-1) \;+\; f(n-2) \end{aligned}$$

[9] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[\,] \qquad \begin{bmatrix} 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Find $f(0)$ and $f(1)$. Find a recurrence relation for $f(n)$. Express $f(n)$ using a matrix power. Find $f(8)$.

(F14 Homework 3) (Solutions)

[1] Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 1 & 3 & 3 & 1 \\ 1 & 4 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

[2] Find the determinant of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 & 1 & 1 \\ 1 & 3 & 1 & 2 & 1 \\ 5 & 1 & 4 & 1 & 1 \\ 1 & 2 & 1 & 3 & 1 \\ 3 & 3 & 3 & 3 & 3 \end{bmatrix}$$

[3] Find the inverse of the matrix

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

[4] Using Cramer's rule, solve for x in the system of equations

$$\begin{bmatrix} 3 & a & 2 \\ 2 & b & 3 \\ 1 & c & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

[5] Find the characteristic equation and a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 1 & 4 \\ 1 & -2 \end{bmatrix}$$

[7] Express $f(n)$ using a matrix power, and find $f(8)$, where

$$f(0) = -1, \quad f(1) = 2$$
$$f(n) = f(n-1) + f(n-2)$$

[8] Express $f(n)$ using a matrix power, and find $f(8)$, where

$$f(0) = 1, \quad f(1) = 1, \quad g(1) = 1$$
$$f(n) = f(n-1) + g(n-1)$$
$$g(n) = f(n-1) + f(n-2)$$

[9] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[\,] \qquad [\,1\,] \qquad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Find $f(0)$ and $f(1)$. Find a recurrence relation for $f(n)$. Express $f(n)$ using a matrix power. Find $f(8)$.

(F14 8:40 Exam 3) (Solutions)

[1] Find the determinant of the matrix

$$\begin{bmatrix} 4 & 6 & 2 & 1 \\ 2 & 1 & 2 & 1 \\ 1 & 6 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

[2] Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

[3] Using Cramer's rule, solve for z in the system of equations

$$\begin{bmatrix} 1 & a & 1 \\ 2 & b & 3 \\ 1 & c & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

[4] Find the characteristic equation and a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 0 & -1 \\ 3 & -4 \end{bmatrix}$$

[5] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[\,] \quad \begin{bmatrix} 2 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Find $f(0)$ and $f(1)$. Find a recurrence relation for $f(n)$. Express $f(n)$ using a matrix power. Find $f(8)$.

(F14 11:40 Exam 3) (Solutions)

[1] Find the determinant of the matrix

$$A = \begin{bmatrix} 3 & 3 & 3 & 3 \\ 1 & 4 & 1 & 1 \\ 1 & 2 & 1 & 4 \\ 2 & 2 & 4 & 2 \end{bmatrix}$$

[2] Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & 2 \end{bmatrix}$$

[3] Using Cramer's rule, solve for y in the system of equations

$$\begin{bmatrix} a & 1 & 2 \\ b & 1 & 3 \\ c & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

[4] Find the characteristic equation and a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 2 & -2 \\ -3 & 1 \end{bmatrix}$$

[5] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[\,] \quad \begin{bmatrix} -2 \end{bmatrix} \quad \begin{bmatrix} -2 & 1 \\ 2 & -2 \end{bmatrix} \quad \begin{bmatrix} -2 & 1 & 0 \\ 2 & -2 & 1 \\ 0 & 2 & -2 \end{bmatrix} \quad \begin{bmatrix} -2 & 1 & 0 & 0 \\ 2 & -2 & 1 & 0 \\ 0 & 2 & -2 & 1 \\ 0 & 0 & 2 & -2 \end{bmatrix} \quad \begin{bmatrix} -2 & 1 & 0 & 0 & 0 \\ 2 & -2 & 1 & 0 & 0 \\ 0 & 2 & -2 & 1 & 0 \\ 0 & 0 & 2 & -2 & 1 \\ 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

Find $f(0)$ and $f(1)$. Find a recurrence relation for $f(n)$. Express $f(n)$ using a matrix power. Find $f(8)$.

(F14 Homework 4) (Solutions)

[1] Find A^n where A is the matrix

$$A = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}$$

(F14 8:40 Final) (Solutions)

[2] Find the 3×3 matrix A that maps the vector $(1, 2, 1)$ to $(3, 6, 3)$, and maps each point on the plane $x+y+z = 0$ to the zero vector.

[3] Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 3 & 2 \\ -2 & -2 \end{bmatrix}$$

(F14 11:40 Final) (Solutions)

[2] Find the 3×3 matrix A that maps the vector $(1, 1, 0)$ to $(2, 2, 0)$, and maps each point on the plane $x+y+z = 0$ to itself.

[3] Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & 1 \\ 1 & 3 & 2 \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$$

(S14 Exam 2) (Solutions)

[3] Find the 3×3 matrix that projects orthogonally onto the plane

$$x + 3y - 2z = 0$$

(S14 Exam 3) (Solutions)

[1] Find the determinant of the matrix

$$\begin{bmatrix} 2 & 2 & 3 & 3 \\ 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 6 \\ 3 & 3 & 4 & 9 \end{bmatrix}$$

[2] Find the inverse of the matrix

$$\begin{bmatrix} 0 & 2 & 1 \\ 3 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix}$$

[3] Find w/y where

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

[4] Find A^n where A is the matrix

$$\begin{bmatrix} 2 & 6 \\ 2 & 3 \end{bmatrix}$$

Your final answer should be in the form

$$A^n = r^n B + s^n C$$

[5] Find $f(n)$, where $f(n)$ is the determinant of the $n \times n$ matrix in the sequence

$$\begin{bmatrix} 5 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 \\ 3 & 5 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 & 0 \\ 3 & 5 & 2 \\ 0 & 3 & 5 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 & 0 & 0 \\ 3 & 5 & 2 & 0 \\ 0 & 3 & 5 & 2 \\ 0 & 0 & 3 & 5 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 & 0 & 0 & 0 \\ 3 & 5 & 2 & 0 & 0 \\ 0 & 3 & 5 & 2 & 0 \\ 0 & 0 & 3 & 5 & 2 \\ 0 & 0 & 0 & 3 & 5 \end{bmatrix}$$

Your final answer should be in the form $f(n) = a\,r^n + b\,s^n$

(S14 Final) (Solutions)

[2] Find the 3×3 matrix that projects orthogonally onto the plane

$$x - z \;=\; 0$$

[3] Find $f(n)$, where $f(n)$ is the determinant of the $n \times n$ matrix in the sequence

$$\begin{bmatrix} 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 2 & 0 \\ 1 & 3 & 2 \\ 0 & 1 & 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 2 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 2 & 0 & 0 & 0 \\ 1 & 3 & 2 & 0 & 0 \\ 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

Your final answer should be in the form $\quad f(n) = a\, r^n \,+\, b\, s^n$

(F13 Exam 2) (Solutions)

[3] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that projects orthogonally onto the subspace V spanned by $(1, -1, 0)$ and $(0, 2, 1)$. Find the matrix A that represents L in standard coordinates.

(F13 Exam 3) (Solutions)

[1] Find the determinant of the matrix

$$\begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 4 & 1 & 1 \\ 2 & 3 & 3 & 2 \\ 1 & 4 & 2 & 3 \end{bmatrix}$$

[2] Find the determinant of the matrix

$$\begin{bmatrix} 2 & 3 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

[3] Find x/y where

$$\begin{bmatrix} a & b & c & d \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

[4] Find the inverse of the matrix

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 0 \\ 1 & 3 & 0 \end{bmatrix}$$

[5] Find A^n where A is the matrix

$$\begin{bmatrix} 2 & -1 \\ -4 & -1 \end{bmatrix}$$

(F13 Final) (Solutions)

[3] Find the determinant of the matrix

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 \end{bmatrix}$$

(S13 8:40 Exam 2) (Solutions)

[3] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that projects orthogonally onto the line

$$x = y = 2z$$

Find the matrix A that represents L in standard coordinates.

(S13 10:10 Exam 2) (Solutions)

[3] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that projects orthogonally onto the subspace

$$x + 2y + z = 0$$

Find the matrix A that represents L in standard coordinates.

(S13 8:40 Exam 3) (Solutions)

[1] Compute the determinant of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 4 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 9 & 1 \end{bmatrix}$$

[2] Find w/z where

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

[3] Compute A^n for the matrix

$$A = \begin{bmatrix} -1 & 1 \\ 5 & 3 \end{bmatrix}$$

(S13 10:10 Exam 3) (Solutions)

[1] Compute the determinant of the matrix

$$A = \begin{bmatrix} 0 & 3 & 2 & 0 \\ 3 & 6 & 9 & 2 \\ 2 & 9 & 6 & 3 \\ 0 & 2 & 3 & 0 \end{bmatrix}$$

[2] Find w/z where

$$\begin{bmatrix} a & b & c & d \\ 1 & 1 & 5 & 1 \\ 1 & 0 & 1 & 1 \\ 3 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

[3] Compute A^n for the matrix

$$A = \begin{bmatrix} -1 & 2 \\ 5 & 2 \end{bmatrix}$$

(S13 Alt Exam 3) (Solutions)

[1] Compute the determinant of the matrix

$$A = \begin{bmatrix} 0 & 2 & 1 & 0 \\ 1 & 3 & 4 & 0 \\ 0 & 4 & 3 & 1 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

[2] Find w/z where

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

[3] Compute A^n for the matrix

$$A = \begin{bmatrix} -1 & 3 \\ 5 & 1 \end{bmatrix}$$

(S13 8:40 Final)

[2] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that projects orthogonally onto the plane

$$x - y = 0$$

Find the matrix A that represents L in standard coordinates.

(S13 10:10 Final)

[2] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that projects orthogonally onto the line

$$x = y = z$$

Find the matrix A that represents L in standard coordinates.

(S13 Alt Final)

[2] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that projects orthogonally onto the plane

$$x + y + z = 0$$

Find the matrix A that represents L in standard coordinates.

(S12 9:10 Exam 2) (Solutions)

[4] Using Cramer's rule, solve for z in

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 3 & 0 & 1 \\ 1 & 3 & 3 & 1 \\ 1 & 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

(S12 11:00 Exam 2) (Solutions)

[4] Find the determinant of the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 & a & 0 \\ 1 & b & 0 & 0 & 0 \\ 0 & 0 & 1 & c & 0 \\ 1 & d & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e \end{bmatrix}.$$

(S12 Practice Final A) (Solutions)

[1] Find A^n where A is the matrix

$$\begin{bmatrix} -1 & 2 \\ 3 & -2 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$\begin{bmatrix} -1 & 3 \\ 3 & -1 \end{bmatrix}$$

(S12 Practice Final B)

[1] Find A^n where A is the matrix

$$\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$\begin{bmatrix} -2 & -3 \\ 1 & 2 \end{bmatrix}$$

(S12 Practice Final C)

[1] Find A^n where A is the matrix

$$\begin{bmatrix} -1 & -3 \\ 2 & 4 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$\begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$$

(S12 Practice Final D)

[1] Find A^n where A is the matrix

$$\begin{bmatrix} -3 & 1 \\ 2 & -2 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

(S12 9:10 Final)

[1] Find A^n where A is the matrix

$$\begin{bmatrix} -2 & 1 \\ 4 & 1 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$$

(S12 11:00 Final)

[1] Find A^n where A is the matrix

$$\begin{bmatrix} 3 & 3 \\ 4 & -1 \end{bmatrix}$$

[2] Find A^n where A is the matrix

$$\begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix}$$

(S11 Exam 2) (Solutions)

[2] Find the determinant of each of the following matrices.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 3 & 4 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 9 \end{bmatrix} \qquad \begin{bmatrix} a & b & c & d \\ a & b+1 & c & d \\ a & b & c+1 & d \\ a & b & c & d+1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 4 \\ 1 & 2 & 6 & 4 \\ 1 & 2 & 3 & 8 \end{bmatrix}$$

[3] Find the inverse of the following matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ a & 1 & 0 & c \\ b & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[4] Let

$$v_1 = (1,0,0), \qquad v_2 = (1,1,0), \qquad v_3 = (0,1,1)$$

Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear map such that

$$L(v_1) = v_2, \qquad L(v_2) = v_3, \qquad L(v_3) = v_1,$$

Find the matrix A (in standard coordinates) that represents the linear map L.

[5] Find the ratio x/y for the solution to the matrix equation

$$\begin{bmatrix} a & d & 1 \\ b & e & 1 \\ c & f & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

[6] Find the determinant of the following 5×5 matrix. What is the determinant for the $n \times n$ case?

$$\begin{bmatrix} x & x^2 & 0 & 0 & 0 \\ 1 & x & x^2 & 0 & 0 \\ 0 & 1 & x & x^2 & 0 \\ 0 & 0 & 1 & x & x^2 \\ 0 & 0 & 0 & 1 & x \end{bmatrix}$$

(S11 Final) (Solutions)

[5] Find the eigenvalues and corresponding eigenvectors of the matrix

$$A \quad = \quad \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$$