F16 8:40 Exam 1 Problem 1
Linear Algebra, Dave Bayer

test1a2p1

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Test 1

Name Solutions Uni

[1] Find the determinant of the matrix

$$\begin{bmatrix} \oplus \\ - \\ + \\ - \\ + \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 & 3 \\ 0 & 1 & 1 & 5 & 5 \\ 2 & 2 & 2 & 2 & 2 \\ 0 & 1 & 1 & 1 & 7 \end{bmatrix}$$

det(A) $= \boxed{-96}$

$$\det(A) = -2 \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 3 & 3 \\ 1 & 1 & 5 & 5 \\ 1 & 1 & 1 & 7 \end{vmatrix} = -2 \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 6 \end{vmatrix}$$

$$= -2 \cdot 2 \cdot 4 \cdot 6 = -16 \cdot 6 = -96$$

F16 8:40 Exam 1 Problem 2
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test1a2p2

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Test 1

[2] Find the inverse to the matrix

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\begin{array}{c|cccc} & 1 & 2 & 0 & 1 \; 2 \\ \hline 3 & 2 & 1 & 3 & 2 \\ 2 & 3 & 1 & 2 & 3 \\ 1 & 2 & 0 & 1 & 2 \\ 3 & 2 & 1 & 3 & 2 \end{array}$$

$$\begin{bmatrix} -1 & -1 & 5 \\ -2 & 1 & 1 \\ 2 & -1 & -4 \end{bmatrix} / -3$$

$$A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & -5 \\ 2 & -1 & -1 \\ -2 & 1 & 4 \end{bmatrix}$$

F16 8:40 Exam 1 Problem 3
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test1a2p3

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Test 1

[3] Using Cramer's rule, solve for x in the system of equations

$$\begin{bmatrix} 1 & a & 0 \\ 2 & b & 1 \\ 4 & c & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$$

$$x = \frac{(4)\,a + (1)\,b + (-1)\,c}{(2)\,a + (1)\,b + (-1)\,c}$$

$$\frac{\begin{vmatrix} 1 & a & 0 \\ 1 & b & 1 \\ 5 & c & 1 \end{vmatrix}}{\begin{vmatrix} 1 & a & 0 \\ 2 & b & 1 \\ 4 & c & 1 \end{vmatrix}} = \frac{-a\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} + b\begin{vmatrix} 1 & 0 \\ 5 & 1 \end{vmatrix} - c\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}}{-a\begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} + b\begin{vmatrix} 1 & 0 \\ 4 & 1 \end{vmatrix} - c\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix}}$$

$$= \frac{4a + b - c}{2a + b - c}$$

check: (choose values for a,b,c where we know the solu)

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 1 \\ 4 & 9 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$$

$$\left.\begin{matrix} a=2 \\ b=3 \\ c=9 \end{matrix}\right\} \Rightarrow x=-1 \qquad \frac{8+3-9}{4+3-9} = \frac{11-9}{7-9} = -1 \checkmark$$

F16 8:40 Exam 1 Problem 4
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test1a2p4

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Test 1

[4] Find a system of eigenvalues and eigenvectors for the matrix A, and find a formula for A^n, where

$$A = \begin{bmatrix} 3 & 1 \\ 4 & 6 \end{bmatrix}$$

$$\lambda_1 + \lambda_2 = 9$$

$$\lambda_1 \lambda_2 = 18 - 4 = 14$$

$$\boxed{2, 7}$$

$$\lambda_1,\ \lambda_2 = \boxed{2},\ \boxed{7}$$

$$v_1,\ v_2 = \begin{bmatrix} \boxed{1} \\ \boxed{-1} \end{bmatrix},\ \begin{bmatrix} \boxed{1} \\ \boxed{4} \end{bmatrix}$$

$$A^n = \frac{\boxed{2^n}}{\boxed{5}} \begin{bmatrix} \boxed{4} & \boxed{-1} \\ \boxed{-4} & \boxed{1} \end{bmatrix} + \frac{\boxed{7^n}}{\boxed{5}} \begin{bmatrix} \boxed{1} & \boxed{1} \\ \boxed{4} & \boxed{4} \end{bmatrix}$$

$$2: \begin{bmatrix} 1 & 1 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 0$$

$$A^n = 2^n \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 4 & -1 \end{bmatrix} / 5 + 7^n \begin{bmatrix} 1 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} / 5$$

$$7: \begin{bmatrix} -4 & 1 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = 0$$

check $n=0$ I

$$\begin{array}{c|c} 4 \quad {}_1 & -1 \quad {}_1 \\ \hline -4 \quad {}_4 & 1 \quad {}_4 \end{array} \ /5 \quad \checkmark$$

$n=1$ A

$$\begin{array}{c|c} 8 \quad {}_7 & -2 \quad {}_7 \\ \hline -8 \quad {}_{28} & 2 \quad {}_{28} \end{array} \ /5 \quad \checkmark$$

test1a2p5

[Reserved for Score]

Test 1

[5] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[\,] \qquad [1] \qquad \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & -1 \\ 0 & 2 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & 0 & 0 \\ 2 & 1 & -1 & 0 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 2 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 2 & 1 & -1 & 0 & 0 \\ 0 & 2 & 1 & -1 & 0 \\ 0 & 0 & 2 & 1 & -1 \\ 0 & 0 & 0 & 2 & 1 \end{bmatrix}$$

Find a recurrence relation for $f(n)$. Express $f(n)$ using a matrix power. Find a formula for $f(n)$.

$$f(n) = (\boxed{1})\, f(n-1) + (\boxed{2})\, f(n-2)$$

$$\begin{bmatrix} f(n) \\ f(n+1) \end{bmatrix} = \begin{bmatrix} \boxed{0} & \boxed{1} \\ \boxed{2} & \boxed{1} \end{bmatrix}^n \begin{bmatrix} f(0) \\ f(1) \end{bmatrix}$$

$$f(n) = \frac{1}{3}(-1)^n + \frac{2}{3}(2)^n$$

$A = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$

$\lambda_1 + \lambda_2 = 1$

$\lambda_1 \lambda_2 = -2$ $\quad \boxed{-1, 2}$

$A + 1I: \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 0$

$A - 2I: \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0$

$A^n = (-1)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 2 & -1 \end{bmatrix} / 3 + 2^n \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} / 3$

$A^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{3}(-1)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \frac{2}{3} 2^n \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\boxed{f(n) = \frac{1}{3}(-1)^n + \frac{2}{3} 2^n}$