

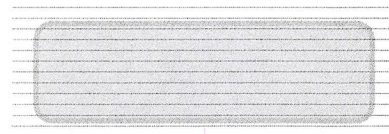


test1a2p1

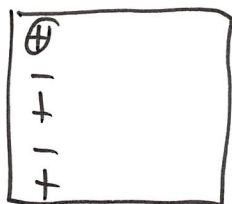
[Reserved for Score]

Test 1

Name Solutions Uni _____



[1] Find the determinant of the matrix



$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 & 3 \\ 0 & 1 & 1 & 5 & 5 \\ 2 & 2 & 2 & 2 & 2 \\ 0 & 1 & 1 & 1 & 7 \end{bmatrix}$$

$$\det(A) = -96$$

$$\det(A) = -2 \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 3 & 3 \\ 1 & 1 & 5 & 5 \\ 1 & 1 & 1 & 7 \end{vmatrix} = -2 \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 6 \end{vmatrix}$$

$$= -2 \cdot 2 \cdot 4 \cdot 6 = -16 \cdot 6 = -96$$



Test 1

[2] Find the inverse to the matrix

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\begin{array}{c|ccc} 1 & 2 & 0 & 1 & 2 \\ 3 & 2 & 1 & 3 & 2 \\ 2 & 3 & 1 & 2 & 3 \\ 1 & 2 & 0 & 1 & 2 \\ 3 & 2 & 1 & 3 & 2 \end{array}$$

$$\begin{bmatrix} -1 & -1 & 5 \\ -2 & 1 & 1 \\ 2 & -1 & -4 \end{bmatrix} \div -3$$

$$A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & -5 \\ 2 & -1 & -1 \\ -2 & 1 & 4 \end{bmatrix}$$



Test 1

[3] Using Cramer's rule, solve for x in the system of equations

$$\begin{bmatrix} 1 & a & 0 \\ 2 & b & 1 \\ 4 & c & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$$

$$\frac{\begin{vmatrix} 1 & a & 0 \\ 1 & b & 1 \\ 5 & c & 1 \end{vmatrix}}{\begin{vmatrix} 1 & a & 0 \\ 2 & b & 1 \\ 4 & c & 1 \end{vmatrix}} = \frac{-a \begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} + b \begin{vmatrix} 1 & 0 \\ 5 & 1 \end{vmatrix} - c \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}}{-a \begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} + b \begin{vmatrix} 1 & 0 \\ 4 & 1 \end{vmatrix} - c \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix}}$$

$$x = \frac{(\boxed{4})a + (\boxed{1})b + (\boxed{-1})c}{(\boxed{2})a + (\boxed{1})b + (\boxed{-1})c}$$

$$= \frac{4a + b - c}{2a + b - c}$$

check: (choose values for a, b, c where we know the solu)

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 1 \\ 4 & 9 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$$

$$\left. \begin{matrix} a=2 \\ b=3 \\ c=9 \end{matrix} \right\} \Rightarrow x = -1$$

$$\frac{8+3-9}{4+3-9} = \frac{11-9}{7-9} = -1 \checkmark$$



Test 1

[4] Find a system of eigenvalues and eigenvectors for the matrix A , and find a formula for A^n , where

$$A = \begin{bmatrix} 3 & 1 \\ 4 & 6 \end{bmatrix}$$

$$\lambda_1 + \lambda_2 = 9$$

$$\lambda_1 \lambda_2 = 18 - 4 = 14$$

$$\boxed{2, 7}$$

$$\begin{aligned} \lambda_1, \lambda_2 &= \boxed{2}, \boxed{7} \\ v_1, v_2 &= \begin{bmatrix} \boxed{1} \\ \boxed{-1} \end{bmatrix}, \begin{bmatrix} \boxed{1} \\ \boxed{4} \end{bmatrix} \\ A^n &= \frac{\boxed{2^n}}{\boxed{5}} \begin{bmatrix} \boxed{4} & \boxed{-1} \\ \boxed{-4} & \boxed{1} \end{bmatrix} + \frac{\boxed{7^n}}{\boxed{5}} \begin{bmatrix} \boxed{1} & \boxed{1} \\ \boxed{4} & \boxed{4} \end{bmatrix} \end{aligned}$$

$$2: \begin{bmatrix} 1 & 1 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 0$$

$$7: \begin{bmatrix} -4 & 1 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = 0$$

$$A^n = 2^n \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 4 & -1 \end{bmatrix} / 5 + 7^n \begin{bmatrix} 1 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} / 5$$

check $n=0$
I $\begin{array}{c|cc} 4 & 1 & -1 \\ -4 & 1 & 4 \end{array} / 5 \quad \checkmark$

$n=1$
A $\begin{array}{c|cc} 8 & 7 & -2 \\ -8 & 2 & 28 \end{array} / 5 \quad \checkmark$



Test 1

[5] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[] \quad [1] \quad \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & -1 \\ 0 & 2 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 & 0 & 0 \\ 2 & 1 & -1 & 0 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 2 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 2 & 1 & -1 & 0 & 0 \\ 0 & 2 & 1 & -1 & 0 \\ 0 & 0 & 2 & 1 & -1 \\ 0 & 0 & 0 & 2 & 1 \end{bmatrix}$$

Find a recurrence relation for $f(n)$. Express $f(n)$ using a matrix power. Find a formula for $f(n)$.

$$A = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

$$\lambda_1 + \lambda_2 = 1$$

$$\lambda_1 \lambda_2 = -2$$

$$\boxed{-1, 2}$$

$$f(n) = \boxed{1} f(n-1) + \boxed{2} f(n-2)$$

$$\begin{bmatrix} f(n) \\ f(n+1) \end{bmatrix} = \begin{bmatrix} \boxed{0} & \boxed{1} \\ \boxed{2} & \boxed{1} \end{bmatrix}^n \begin{bmatrix} f(0) \\ f(1) \end{bmatrix}$$

$$f(n) = \frac{1}{3} (-1)^n + \frac{2}{3} (2)^n$$

$$A + 1I: \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 0$$

$$A - 2I: \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0$$

$$A^n = (-1)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} \frac{1}{3} + 2^n \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} \frac{1}{3}$$

$$A^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{3} (-1)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \frac{2}{3} 2^n \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\boxed{f(n) = \frac{1}{3} (-1)^n + \frac{2}{3} 2^n}$$