F16 10:10 Exam 1 Problem 1
Linear Algebra, Dave Bayer

test1b1p1

[Reserved for Score]

Test 1

Name ______________________________ Uni ______________

[1] Find the general solution to the following system of equations.

$$\begin{bmatrix} 5 & 7 & 1 & 2 \\ 3 & 4 & 1 & 1 \\ 2 & 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} =$$

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Linear Algebra, Dave Bayer

test1b1p2

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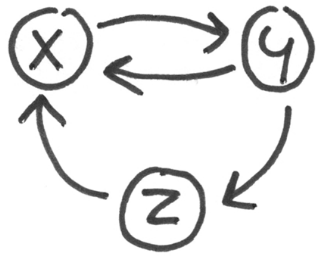
Test 1

[2] Using matrix multiplication, count the number of paths of length 16 from x to x.



number of paths $=$

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test1b1p3

[Reserved for Score]

Test 1

[3] Find the intersection of the following two affine subspaces of $\mathbb{R}^3$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}, \qquad 2x + y - z = 2$$

$$\boxed{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \qquad\qquad}$$

test1b1p4

[Reserved for Score]

Test 1

[4] By least squares, find the equation of the form $y = ax^2 + b$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$$

(Note that x is *squared* in $y = ax^2 + b$.)

$$y = \boxed{}\, x^2 + \boxed{}$$

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test1b1p5

[Reserved for Score]

Test 1

[5] Let V be the vector space $\mathbb{R}^3$, equipped with the inner product

$$\langle (a,b,c),(d,e,f)\rangle \;=\; \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} d \\ e \\ f \end{bmatrix}$$

Using this inner product to define orthogonality, find an orthogonal basis for the plane defined by the equation

$$x + y = 0$$

Extend this basis to an orthogonal basis for $\mathbb{R}^3$.

$v_1 = \begin{bmatrix} \quad & \quad & \quad \end{bmatrix}$

$v_2 = \begin{bmatrix} \quad & \quad & \quad \end{bmatrix}$

$v_3 = \begin{bmatrix} \quad & \quad & \quad \end{bmatrix}$