F15 Homework 2
Linear Algebra, Dave Bayer

[1] Let v_1, v_2, … v_r be a set of vectors that spans a subspace W of a vector space V. Prove that one can choose a subset of these vectors that forms a basis for W, and that this basis can be extended to a basis for V. Demonstrate this procedure on the vectors

$$(1,-1,0,0) \quad (1,0,-1,0) \quad (1,0,0,-1) \quad (0,1,-1,0) \quad (0,1,0,-1) \quad (0,0,1,-1)$$

[2] Find the 3×3 matrix that vanishes on the plane $3x - 2y + z = 0$, and maps the vector $(1,0,0)$ to itself.

[3] Find the 3×3 matrix that vanishes on the vector $(1,0,2)$, and stretches each vector in the plane $x+y=0$ by a factor of 3.

[4] Find the 3×3 matrix that projects orthogonally onto the line

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} t$$

[5] Find the 3×3 matrix that projects orthogonally onto the plane

$$x - 2y + z = 0$$

[6] Find the row space and the column space of the matrix

$$\begin{bmatrix} 0 & 1 & 1 & 2 & 3 \\ 1 & 1 & 2 & 3 & 5 \\ 1 & 2 & 3 & 5 & 8 \end{bmatrix}$$

[7] By least squares, find the equation of the form $y = ax + b$ that best fits the data

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$$

[8] Find an orthogonal basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors

$$(1,2,0,0) \quad (0,1,2,0) \quad (0,0,1,2) \quad (1,0,-4,0) \quad (0,1,0,-4)$$

Extend this basis to an orthogonal basis for $\mathbb{R}^4$.

[9] Let V be the vector space of all polynomials of degree $\leqslant 2$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace consisting of those polynomials $f(x)$ such that $f(0) = 0$. Find the orthogonal projection of the polynomial $x + 2$ onto the subspace W, with respect to the inner product

$$\langle f, g \rangle = \int_0^1 f(x)g(x)\,dx$$