F15 Final Exam Problem 1
Linear Algebra, Dave Bayer

test1a4p1

[Reserved for Score]

Test 1

Name _____ Uni _____

[1] Solve the following system of equations.

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} =$$

F15 Final Exam Problem 2
Linear Algebra, Dave Bayer

test1a4p2

[Reserved for Score]

Test 1

[2] Find the 3×3 matrix A such that

$$A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad A \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad A \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

$$A = \frac{1}{\square} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

F15 Final Exam Problem 3
Linear Algebra, Dave Bayer

test1a4p3

[Reserved for Score]

Test 1

[3] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$\begin{bmatrix} 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Find $f(8)$.

$f(8) =$

F15 Final Exam Problem 4
Linear Algebra, Dave Bayer

test1a4p4

[Reserved for Score]

Test 1

[4] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & -3 \\ -2 & 0 \end{bmatrix}$$

$$e^{At} = \frac{\square}{\square}\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

F15 Final Exam Problem 5
Linear Algebra, Dave Bayer

test1a4p5

[Reserved for Score]

Test 1

[5] Find A^n where A is the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 2 \end{bmatrix}$$

$$A^n = \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

F15 Final Exam Problem 6
Linear Algebra, Dave Bayer

test1a4p6

[Reserved for Score]

Test 1

[6] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$y = \frac{\square}{\square} \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} + \frac{\square}{\square} \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} + \frac{\square}{\square} \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$$

F15 Final Exam Problem 7
Linear Algebra, Dave Bayer

test1a4p7

[Reserved for Score]

Test 1

[7] Express the quadratic form

$$2x^2 + 2y^2 - 2xz + 2yz + 3z^2$$

as a sum of squares of othogonal linear forms.

$$\square\left(\square\right)^2 + \square\left(\square\right)^2 + \square\left(\square\right)^2$$

F15 Final Exam Problem 8
Linear Algebra, Dave Bayer

test1a4p8

[Reserved for Score]

Test 1

[8] Solve for z in the system of differential equations

$$\begin{aligned} y'' &= 2y' + y + z \\ z' &= -2y' + 2y + z \end{aligned}$$

where

$$y(0) = y'(0) = 0, \qquad z(0) = 1$$

$z(t) =$