F15 Exam 3
Linear Algebra, Dave Bayer

[1] Find the determinant of the matrix

$$A = \begin{bmatrix} 3 & 6 & 6 & 1 & 1 \\ 1 & 3 & 6 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 1 & 1 & 6 \\ 1 & 1 & 1 & 1 & 3 \end{bmatrix}$$

[2] Using Cramer's rule, solve for z in the system of equations

$$\begin{bmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

[3] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$\begin{bmatrix} 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 2 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 2 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Find $f(10)$.

[4] Find a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 4 & 6 \\ 1 & 5 \end{bmatrix}$$

[5] Find a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 1 & -1 & 2 \end{bmatrix}$$