F14 Practice 3 Problem 1
Linear Algebra, Dave Bayer

[1] Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 3 & 4 & 1 & 5 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$\det(A) \;=\; \boxed{}$

F14 Practice 3 Problem 2
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[2] Find the determinant of the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 0 \\ 1 & 1 & 3 & 1 & 2 \\ 1 & 1 & 1 & 3 & 0 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix}$$

$\det(A) = \square$

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[3] Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\square} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

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[4] Using Cramer's rule, solve for x in the system of equations

$$\begin{bmatrix} 3 & 1 & a \\ 2 & 1 & b \\ 1 & 1 & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 2 \end{bmatrix}$$

$$x = \frac{(\square)\, a + (\square)\, b + (\square)\, c}{(\square)\, a + (\square)\, b + (\square)\, c}$$

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[5] Find the characteristic equation and a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\lambda^2 + (\square)\,\lambda + (\square) = 0$$

$$\lambda_1,\ \lambda_2 = \square,\ \square$$

$$v_1,\ v_2 = \begin{bmatrix} \square \\ \square \end{bmatrix},\ \begin{bmatrix} \square \\ \square \end{bmatrix}$$

F14 Practice 3 Problem 6
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[6] Find the characteristic equation and a system of eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ 0 & -2 & 2 \end{bmatrix}$$

$$\lambda^3 + (\square)\,\lambda^2 + (\square)\,\lambda + (\square) = 0$$

$$\lambda_1,\ \lambda_2,\ \lambda_3 = \square,\ \square,\ \square$$

$$v_1,\ v_2,\ v_3 = \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix},\ \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix},\ \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$$

F14 Practice 3 Problem 7
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[7] Express $f(n)$ using a matrix power, and find $f(8)$, where

$$f(0) = 1, \quad f(1) = 2$$
$$f(n) \;=\; 2\,f(n-1) \;-\; f(n-2)$$

$$\begin{bmatrix} f(n) \\ f(n+1) \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}^n \begin{bmatrix} f(0) \\ f(1) \end{bmatrix}$$

$$f(8) \;=\; \square$$

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[8] Express $f(n)$ using a matrix power, and find $f(8)$, where

$$f(0) = 1, \quad f(1) = 1, \quad g(1) = 2$$
$$f(n) \;=\; f(n-1) \;+\; g(n-1) \;+\; f(n-2)$$
$$g(n) \;=\; f(n-1) \;-\; g(n-1) \;+\; f(n-2)$$

$$\begin{bmatrix} f(n) \\ f(n+1) \\ g(n+1) \end{bmatrix} = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}^n \begin{bmatrix} f(0) \\ f(1) \\ g(1) \end{bmatrix}$$

$$f(8) \;=\; \square$$

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[9] Let $f(n)$ be the determinant of the $n \times n$ matrix in the sequence

$$[\,] \qquad \begin{bmatrix} 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Find $f(0)$ and $f(1)$. Find a recurrence relation for $f(n)$. Express $f(n)$ using a matrix power. Find $f(8)$.

$$f(0) = \square \qquad f(1) = \square$$

$$f(n) = (\square)\, f(n-1) + (\square)\, f(n-2)$$

$$\begin{bmatrix} f(n) \\ f(n+1) \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}^n \begin{bmatrix} f(0) \\ f(1) \end{bmatrix}$$

$$f(8) = \square$$