F14 11:40 Final Exam Problem 1
Linear Algebra, Dave Bayer

[1] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} + \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} t$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} t$$

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[2] Find the 3×3 matrix A that maps the vector $(1,1,0)$ to $(2,2,0)$, and maps each point on the plane $x+y+z=0$ to itself.

$$A = \frac{1}{\square} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

$$A \;=\; \frac{1}{2}\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

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[3] Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & 1 \\ 1 & 3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\square} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

$$A = \frac{1}{3} \begin{bmatrix} -3 & 6 & 0 \\ -1 & 0 & 1 \\ 3 & -3 & 0 \end{bmatrix}$$

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[4] Find A^n where A is the matrix

$$A = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$$

$$A^n = \frac{(\square)^n}{\square} \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + \frac{(\square)^n}{\square} \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

$$\lambda = -2, 1 \qquad A^n = \frac{(-2)^n}{3} \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$$

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[5] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$y = \frac{\square}{\square} \begin{bmatrix} \square \\ \square \end{bmatrix} + \frac{\square}{\square} \begin{bmatrix} \square \\ \square \end{bmatrix}$$

$$\lambda = -1, 3 \qquad e^{At} = \frac{e^{-t}}{4} \begin{bmatrix} 1 & -3 \\ -1 & 3 \end{bmatrix} + \frac{e^{3t}}{4} \begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix} \qquad y = \frac{e^{-t}}{4} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \frac{e^{3t}}{4} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

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[6] Find e^{At} where A is the matrix

$$A \;=\; \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

$$e^{At} \;=\; \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

$$\lambda = -1, 1, 3 \qquad e^{At} \;=\; \frac{e^{-t}}{4}\begin{bmatrix} 0 & -1 & 1 \\ 0 & 2 & -2 \\ 0 & -2 & 2 \end{bmatrix} + \frac{e^{t}}{2}\begin{bmatrix} 2 & -1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{e^{3t}}{4}\begin{bmatrix} 0 & 3 & 3 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{bmatrix}$$

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[7] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$y = \frac{\square}{\square}\begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} + \frac{\square}{\square}\begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$$

$$\lambda = 3, 1, 1 \qquad e^{At} = \frac{e^{3t}}{4}\begin{bmatrix} 2 & 2 & 3 \\ 2 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} + \frac{e^{t}}{4}\begin{bmatrix} 2 & -2 & -3 \\ -2 & 2 & -3 \\ 0 & 0 & 4 \end{bmatrix} + \frac{te^{t}}{2}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$y = \frac{e^{3t}}{4}\begin{bmatrix} 7 \\ 7 \\ 0 \end{bmatrix} + \frac{e^{t}}{4}\begin{bmatrix} -3 \\ -3 \\ 4 \end{bmatrix} + \frac{te^{t}}{2}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

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[8] Express the quadratic form

$$2x^2 + 2xy + 3y^2 - 2yz + 2z^2$$

as a sum of squares of orthogonal linear forms.

$$\square\left(\square\right)^2 + \square\left(\square\right)^2 + \square\left(\square\right)^2$$

$$\lambda = 1, 2, 4 \qquad A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} + \frac{2}{3}\begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ -1 & -2 & 1 \end{bmatrix}$$

$$\frac{1}{3}(x - y - z)^2 + (x + z)^2 + \frac{2}{3}(x + 2y - z)^2$$