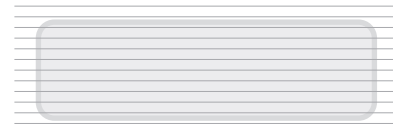


Exam 01

Name \_\_\_\_\_ Uni \_\_\_\_\_

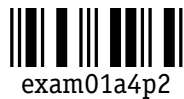


[1] Find the intersection of the following two affine subspaces of  $\mathbb{R}^4$ .

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}$$

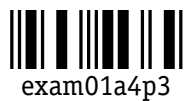
$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} + \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} t$$



**Exam 01**

[2] Find the  $3 \times 3$  matrix  $A$  that maps the vector  $(1, 1, 0)$  to  $(2, 2, 0)$ , and maps each point on the plane  $x + y + z = 0$  to itself.

$$A = \frac{1}{\boxed{\phantom{000}}} \begin{bmatrix} \boxed{\phantom{000}} & \boxed{\phantom{000}} & \boxed{\phantom{000}} \\ \boxed{\phantom{000}} & \boxed{\phantom{000}} & \boxed{\phantom{000}} \\ \boxed{\phantom{000}} & \boxed{\phantom{000}} & \boxed{\phantom{000}} \end{bmatrix}$$

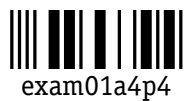


Exam 01

[3] Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & 1 \\ 1 & 3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\boxed{\phantom{000}}} \begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

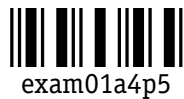


Exam 01

[4] Find  $A^n$  where  $A$  is the matrix

$$A = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$$

$$A^n = \frac{\left(\begin{array}{c} \square \\ \square \end{array}\right)^n}{\square} \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + \frac{\left(\begin{array}{c} \square \\ \square \end{array}\right)^n}{\square} \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

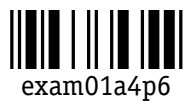


Exam 01

[5] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$y = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \begin{bmatrix} \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \end{bmatrix} + \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \begin{bmatrix} \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \end{bmatrix}$$



Exam 01

[6] Find  $e^{At}$  where  $A$  is the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

$$e^{At} = \frac{\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}}{\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}}{\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} + \frac{\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}}{\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}} \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

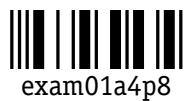


Exam 01

[7] Solve the differential equation  $y' = Ay$  where

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$y = \frac{\begin{bmatrix} \phantom{0} \\ \phantom{0} \end{bmatrix}}{\begin{bmatrix} \phantom{0} \\ \phantom{0} \end{bmatrix}} \begin{bmatrix} \phantom{0} \\ \phantom{0} \\ \phantom{0} \end{bmatrix} + \frac{\begin{bmatrix} \phantom{0} \\ \phantom{0} \end{bmatrix}}{\begin{bmatrix} \phantom{0} \\ \phantom{0} \end{bmatrix}} \begin{bmatrix} \phantom{0} \\ \phantom{0} \\ \phantom{0} \end{bmatrix} + \frac{\begin{bmatrix} \phantom{0} \\ \phantom{0} \end{bmatrix}}{\begin{bmatrix} \phantom{0} \\ \phantom{0} \end{bmatrix}} \begin{bmatrix} \phantom{0} \\ \phantom{0} \\ \phantom{0} \end{bmatrix}$$



Exam 01

[8] Express the quadratic form

$$2x^2 + 2xy + 3y^2 - 2yz + 2z^2$$

as a sum of squares of orthogonal linear forms.

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