F14 11:40 Exam 2
Linear Algebra, Dave Bayer

[1] Find the 3×3 matrix which maps the vector $(1,1,1)$ to $(2,2,2)$, and maps each point on the plane $x + y + z = 0$ to itself.

[2] Find a basis for the row space and a basis for the column space of the matrix

$$\begin{bmatrix} -1 & -1 & 0 & -2 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 & -2 \\ -1 & -1 & 0 & -2 & 1 \end{bmatrix}$$

[3] Find the 3×3 matrix that projects orthogonally onto the plane

$$x + y - 2z = 0$$

[4] Find an orthogonal basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors

$$(-1,1,0,-1) \qquad (-1,0,1,-1) \qquad (0,1,-1,0) \qquad (-2,1,1,-2) \qquad (1,1,-2,1)$$

Extend this basis to an orthogonal basis for $\mathbb{R}^4$.

[5] Let V be the vector space of all polynomials of degree $\leqslant 2$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace of V consisting of those polynomials $f(x)$ such that the derivative $f'(0) = 0$.

Find the orthogonal projection of the polynomial x onto the subspace W, with respect to the inner product

$$\langle f, g \rangle = \int_0^1 f(x)g(x)\,dx$$