F14 11:40 Exam 1 Problem 1
Linear Algebra, Dave Bayer

[1] Solve the following system of equations.

$$\begin{bmatrix} 3 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 4 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \\ -2 \\ -1 \end{bmatrix} t$$

Check:

$$\begin{bmatrix} 3 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 4 & 1 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \\ -2 \\ -1 \end{bmatrix} t \right) = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} t = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 3 & 1 & 0 & 0 & 3 \\ 2 & 0 & 1 & 0 & 2 \\ 4 & 1 & 0 & 1 & 4 \end{array}\right] \xrightarrow[\textcircled{3} \leftarrow \textcircled{3} - \textcircled{1}]{} \left[\begin{array}{cccc|c} 3 & 1 & 0 & 0 & 3 \\ 2 & 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 1 & 1 \end{array}\right]$$

$$\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} w + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} y + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} z = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

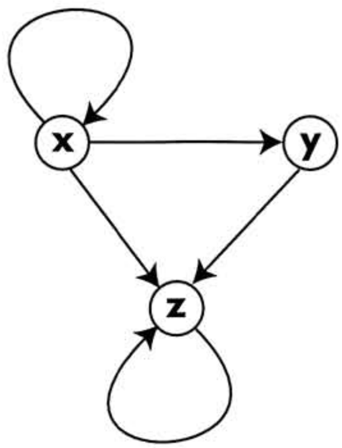
$$\boxed{\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \\ -2 \\ -1 \end{bmatrix} t}$$

homogeneous solution $\mapsto \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

particular solution $\mapsto \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$

F14 11:40 Exam 1 Problem 2
Linear Algebra, Dave Bayer

[2] Using matrix multiplication, count the number of paths of length eight from x to z.



---

15 paths $\quad A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 3 & 1 & 1 \end{bmatrix}, \quad A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 7 & 1 & 1 \end{bmatrix}, \quad A^8 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 15 & 1 & 1 \end{bmatrix}$

$$\text{out}\quad \begin{array}{c c} & \text{in} \\ & \begin{array}{ccc} x & y & z \end{array} \\ \begin{array}{c} x \\ y \\ z \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \end{array} = A$$

(5 entries = 5 edges)

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 7 & 1 & 1 \end{bmatrix}$$

$$A^8 = \begin{array}{cc} & x \\ & \\ z & 15 \end{array}$$

$(7,1,1)\cdot(1,1,7) = 7+1+7$

$\boxed{\text{15 paths}}$

F14 11:40 Exam 1 Problem 3
Linear Algebra, Dave Bayer

[3] Express A as a product of elementary matrices, where

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{1 \leftarrow 1 - 2} \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{1 \leftrightarrow 2} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{1 \leftarrow \frac{1}{3} 1} \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & \\ 1 & 0 & \\ & & 1 \end{bmatrix} \qquad \begin{bmatrix} 3 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

$$\boxed{A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 3 & 1 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

do not include identity matrix
many different solutions are correct

F14 11:40 Exam 1 Problem 4
Linear Algebra, Dave Bayer

[4] Find the matrix A such that

$$A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Check:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \\ \hline 1 & 1 & 3 \\ 1 & 2 & 5 \\ 0 & 1 & 3 \end{array}\right] \xrightarrow[(3) \leftarrow (3) - 2(2)]{} \left[\begin{array}{ccc} 1 & & \\ & 1 & \\ & & 1 \\ \hline 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{array}\right]$$

column reduction
compare with transposing and row reduction
numbers exactly the same

$$\boxed{A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}}$$

check: $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix}$ ☑

always check your work
it will eventually become how you understand everything

F14 11:40 Exam 1 Problem 5
Linear Algebra, Dave Bayer

[5] Find the intersection of the following two affine subspaces of $\mathbb{R}^4$.

$$\begin{bmatrix} 1 & 1 & 1 & -2 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

check

☑ in this space

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}$$

☑ in this space

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} t$$

$$\begin{bmatrix} 1 & 1 & 1 & -2 \\ 1 & 0 & 1 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} t$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} t \right) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ -2 \\ 0 \\ -1 \end{bmatrix} t$$

$$\boxed{\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} t}$$

flip sign, why not.