Final Exam

Linear Algebra, Dave Bayer, December 19, 2013

[1] Find the intersection of the following two affine subspaces of $\mathbb{R}^3$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix}$$

[2] Find an orthogonal basis for the subspace of $\mathbb{R}^4$ defined by the equation $w+x-y-z=0$. Extend this basis to a orthogonal basis for $\mathbb{R}^4$.

[3] Find the determinant of the matrix

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 \end{bmatrix}$$

[4] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

[5] Express the quadratic form

$$-4xy + 3y^2$$

as a sum of squares of othogonal linear forms.

[6] Solve the recurrence relation

$$f(0) = a, \quad f(1) = b, \quad f(n) = 3\,f(n-1) - 2\,f(n-2)$$

[7] Find e^{At} where A is the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

[8] Solve the differential equation $y' = Ay$ where

$$A = \begin{bmatrix} -2 & 2 & -1 \\ -1 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$