Exam 2

Linear Algebra, Dave Bayer, October 22, 2013

[1] Find a basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors

$$(2,0,1,0),\ (2,0,0,1),\ (0,2,1,0),\ (0,2,0,1)$$

Extend this basis to a basis for $\mathbb{R}^4$.

[2] By least squares, find the equation of the form $y = ax + b$ which best fits the data

$$(x_1, y_1) = (-1, 0), \quad (x_2, y_2) = (0, 0), \quad (x_3, y_3) = (1, 0), \quad (x_4, y_4) = (2, 1)$$

[3] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which projects orthogonally onto the subspace V spanned by $(1, -1, 0)$ and $(0, 2, 1)$. Find the matrix A which represents L in standard coordinates.

[4] Let V be the vector space of all polynomials of degree $\leqslant 2$ in the variable x with coefficients in $\mathbb{R}$. Let W be the subspace of polynomials satisfying $f(2) = 0$. Find an orthogonal basis for W with respect to the inner product

$$\langle f, g \rangle \ = \ \int_0^1 f(x) g(x)\, dx$$

[5] Find an orthogonal basis for the subspace of $\mathbb{R}^4$ defined by the equation $w + x - 2y - 2z = 0$. Extend this basis to a orthogonal basis for $\mathbb{R}^4$.