Exam 2

Linear Algebra, Dave Bayer, November 9, 2000

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[1] Let

$$\mathbf{v}_1 = (1,-1,0,0), \quad \mathbf{v}_2 = (-1,1,0,0), \quad \mathbf{v}_3 = (1,-1,1,-1), \quad \mathbf{v}_4 = (-1,1,1,1).$$

Find a basis for the subspace $V \subset \mathbb{R}^4$ spanned by $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$.

[2] Let A be the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \end{bmatrix}.$$

Compute the row space and column space of A.

[3] Let V be the vector space of all polynomials $f(x)$ of degree ≤ 3. Let $W \subset V$ be the set of all polynomials f in V which satisfy $f'(1) = 0$. Show that W is a subspace of V. Find a basis for W. Extend this basis to a basis for V.

[4] Let $\mathbf{v}_1 = (2,1)$ and $\mathbf{v}_2 = (1,2)$. Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that

$$L(\mathbf{v}_1) = \mathbf{v}_1, \quad L(\mathbf{v}_2) = 2\mathbf{v}_2.$$

Find a matrix that represents L with respect to the usual basis $\mathbf{e}_1 = (1,0)$, $\mathbf{e}_2 = (0,1)$.

[5] Let

$$\mathbf{v}_1 = (1,1,0), \quad \mathbf{v}_2 = (1,0,1), \quad \mathbf{v}_3 = (0,1,1).$$

Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation such that

$$L(\mathbf{v}_1) = \mathbf{v}_3, \quad L(\mathbf{v}_2) = \mathbf{v}_1, \quad L(\mathbf{v}_3) = \mathbf{v}_2.$$

Find a matrix that represents L with respect to the usual basis

$$\mathbf{e}_1 = (1,0,0), \quad \mathbf{e}_2 = (0,1,0), \quad \mathbf{e}_3 = (0,0,1).$$