Practice Exam 2

Linear Algebra, Dave Bayer, November 2, 2000

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[1] Let

$$\mathbf{v}_1 = (-1, 2, -1, 0), \quad \mathbf{v}_2 = (-1, 1, 1, -1), \quad \mathbf{v}_3 = (0, -1, 2, -1).$$

Find a basis for the subspace $V \subset \mathbb{R}^4$ spanned by $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$. Extend this basis to a basis for $\mathbb{R}^4$.

[2] Let A be the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 2 & 0 & -2 & 2 \\ 3 & 0 & -3 & 3 \\ 4 & 0 & -4 & 0 \end{bmatrix}.$$

Compute the row space and column space of A.

[3] Let V be the vector space of all twice-differentiable functions $f : \mathbb{R} \to \mathbb{R}$. Let $W \subset V$ be the set of all functions in V which satisfy the differentiable equation $f'' = f$. Show that W is a subspace of V.

[4] Let $\mathbf{v}_1 = (1, 1)$ and $\mathbf{v}_2 = (-1, 1)$. Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that

$$L(\mathbf{v}_1) = 2\,\mathbf{v}_1, \quad L(\mathbf{v}_2) = \mathbf{v}_1 + \mathbf{v}_2.$$

Find a matrix that represents L with respect to the usual basis $\mathbf{e}_1 = (1, 0)$, $\mathbf{e}_2 = (0, 1)$.

[5] Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation such that $L(\mathbf{v}) = \mathbf{v}$ for any vector $\mathbf{v}$ in the plane $x+y+z = 0$, and $L(\mathbf{v}) = \mathbf{0}$ for any vector $\mathbf{v}$ in the line $x = y = z$. Find a matrix that represents L with respect to the usual basis $\mathbf{e}_1 = (1, 0, 0)$, $\mathbf{e}_2 = (0, 1, 0)$, $\mathbf{e}_3 = (0, 0, 1)$.