Final Exam

Linear Algebra, Dave Bayer, May 8, 2001

Name: ____________________

ID: ____________________ **School:** ____________________

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