Complex analysis, lecture 4: syllabus

Fall 2025

Time and location: 9 - 10 AM Mondays, Wednesdays, and Fridays in Etcheverry 3111
Instructor: Avi Zeff, email avizeff@berkeley.edu
Office hours: TBD
TAs: TBD

Welcome to complex analysis! In this course, we will study what is widely considered to be some of the most beautiful mathematics you are likely to encounter in an undergraduate course, if not beyond. Complex analysis is full of miraculous results with applications to geometry, algebra, and number theory, among others, as well as to physics, engineering, and many other fields.

Summary

The material for the course can be grouped into four main areas:

- holomorphic (i.e. complex analytic) functions and their basic properties;
- complex integration and its applications;
- series methods and the residue theorem;
- further topics towards complex analysis and geometry.

By the end of the course, students should have a good working understanding of holomorphic functions; understand the meaning of and be able to compute complex integrals, and understand their utility in complex analysis; be able to use the residue theorem to compute complex and real integrals that would otherwise be impossible; and be able to apply complex analysis to areas such as hyperbolic and complex geometry.

A more detailed course outline can be found at the end of this document.

Prerequisites

The official prerequisite for this course is Math 104, Introduction to Analysis. This will not be rigorously enforced: if you have not taken Math 104 but feel you have sufficient background to do well in this class, I will trust your judgment. What is really needed is a good understanding of calculus, some familiarity with analysis, and a certain level of mathematical maturity. If you are not sure whether you have the prerequisites, email me or come to my office hours.

Textbook

We will follow **Complex Analysis** by Theodore Gamelin. You do not need to purchase it: you may find it helpful to read relevant sections of the book before class (readings will be suggested), but lecture notes will also be posted, and any problems drawn from the book will also be posted on the course website.

Course structure

This course will be taught via "standards-based learning," the central idea of which is that there is a set of objectives which you are here to learn, and the class should be taught in such a way as to optimize the amount of these (both in number and degree) that you learn by the end of the semester, and graded based on how many you have learned to a satisfactory standard. Concretely, this means that both the class structure and grading may be different from what you are used to:

- Your grade will be determined by the number of objectives for which you have demonstrated achievement. Higher grades also require a certain number of "challenge points" (see below for a detailed table).
- Achievement of objectives will be assessed based on in-class exams. To earn credit for an objective, you will need to successfully solve two problems on an exam on that topic.
- Approximately a week after exams are graded, there will be an in-class "retest" with more questions on the same topics, as well as on any earlier ones you may have missed. Successful solutions to problems on retests will count just as well as those on exams, so in a very real sense incorrect solutions don't count: you just need to be able to solve enough problems on each topic over the course of the semester.

Each problem will be graded either S (successful), P (partially successful), or N (not yet successful), with comments to help you improve your future work as needed. Your level of achievement on each objective will be the lowest mark which you have achieved on at least two relevant problems; you are considered to have achieved an objective once your level on it reaches S.

Examples of S-level, P-level, and N-level work are posted on the website. Work which is significantly incomplete, does not address the question, or is illegible to the grader will receive a mark of N, and may receive fewer or no comments.

Challenge points

In addition to the objectives, higher grades in this class also require you to earn a certain number of challenge points over the course of the class. You can earn challenge points by solving challenge problems on exams or retests; these are harder problems which may require you to use the tools and concepts you learn to go beyond what was illustrated in class. There may also be other opportunities to earn challenge points.

It is not expected that anyone will earn every challenge point available; there will be many more points available than are necessary to earn an A+. You should prioritize demonstrating achievement of the objectives over earning challenge points, as you can still achieve a good grade in this class with few or no challenge points, but cannot without achieving most of the objectives. On retests, challenge problems may be available only if you are making sufficient progress on the objectives.

Exams

There will be three midterms and a final exam (tentative dates are on the schedule below, and any changes will be announced). These are closed-book, with no calculators or outside resources. Each exam will include problems on any objective which you have not yet achieved, so if you do poorly on the first exam, you will have more chances on the later ones. However, the more objectives remain, the less time you will have per problem.

For any exam problem on which you are dissatisfied with your grade, you should review the feedback carefully and target that objective on the retest.

Exam problems will be simpler than homework problems, so that they can be solved more quickly, but especially on the later midterms and exams many students will still have more problems on their exams than can reasonably be solved in the allotted time. You should correctly solve as many as you can and defer the rest to retests or future exams.

The final exam will have the same format as the midterm exams, except for being longer, and serves as your last chance to demonstrate each objective. Since some objectives will not yet have been assessed on a midterm, it will also be the only opportunity to demonstrate achievement on them; as such it will include extra problems on these objectives, to give you more opportunities for success.

Homework

Homework is assigned weekly, typically due on Mondays by the end of the day. You should expect to spend between 3-9 hours on these. Collaboration is encouraged, but everyone should write their own solutions; **write on your homeworks anyone you have worked with**. The contribution from each collaborator should be roughly equal: if you find yourself frequently doing more or less than your collaborators, consider finding a different group.

Homework is marked (using the S/P/N scale) with feedback but **not graded**: these marks are to help you gauge your mastery of each objective, but do not directly affect your grade. You may revise and resubmit for more feedback, though graders will give priority to new submissions.

Though homework doesn't directly count toward your course grade, skipping it is strongly discouraged: it is your main active learning opportunity, and your demonstrated progress on homework determines which challenge problems appear on your exams, so skipping homeworks effectively locks you into a lower grade.

We will attempt to have all homework graded within one week of its submission. Late homework may be graded late.

Grading

Your grade will be determined by the number of objectives you achieve, together with challenge points. There are 14 objectives, listed below. All of them are important (or else we would not spend time on them!), but some can be considered to make up the heart of the class; these have been marked as (CORE) below. In order to pass the class (i.e. with a C-) you must achieve all four of these core objectives, up to some flexibility as detailed below.

An objective at level P counts as half an objective; so for example if you have achieved eight objectives to level S, three to level P, and one to level N, then you would be considered to have $8 + 3 \cdot \frac{1}{2} = 9.5$ objectives. Core objectives must be achieved to level S, except for C- or D grades, for which e.g. two core objectives to level S and two to level P would count for $2 + \frac{2}{2} = 3$ core objectives and thus suffice.

A- or B-level grades also require varying numbers of challenge points in addition to meeting objective totals. Thus the grade requirements are as follows:

Grade	**Objectives achieved**	**Challenge points**
A+	14, including all core objectives	≥ 30
A	≥ 13, including all core objectives	≥ 20
A-	≥ 12, including all core objectives	≥ 13
B+	≥ 11, including all core objectives	≥ 8
B	≥ 10, including all core objectives	≥ 5
B-	≥ 9, including all core objectives	≥ 2
C+	≥ 8, including all core objectives	≥ 0
C	≥ 7, including all core objectives	≥ 0
C-	≥ 6, including all core objectives, OR ≥ 8, including ≥ 3 core objectives	≥ 0
D	≥ 6, including ≥ 3 core objectives	≥ 0

Your grade will be the highest for which you've fulfilled both requirements. A grade of F will be assigned if the requirements for a D are not met. For example, if you've achieved 9.5 objectives as above, including all core objectives to level S, and earned 6 challenge points, your grade would be a B-, as that is the highest grade for which you've fulfilled the requirements.

I reserve the right to modify this grading scheme over the course of the semester, but only ever in your favor (e.g. I could change the requirements for a B+ to only requiring 6 challenge points instead of 8, but will not change it to requiring 10).

The objectives are as follows.

(1) Complex numbers and algebra: you understand and can manipulate and use complex numbers and algebraic expressions involving them. (CORE)

(2) Branch points/cuts and Riemann surfaces: you understand how to use branch cuts to make functions single-valued on a given domain, and how to use Riemann surfaces to define single-valued functions on these surfaces in place of multi-valued functions on a given domain.

(3) Analytic functions and the Cauchy–Riemann equations: you know the definition of an analytic function, and can derive and apply the Cauchy–Riemann equations to check if a given function is analytic. (CORE)

(4) Harmonic functions and conformal mappings: you know the definition of harmonic functions and conformal mappings, can check if and where functions are harmonic or conformal and find harmonic conjugates and conformal maps between given domains,

and understand the relationships with analytic functions.

(5) Complex integration: you can set up and evaluate contour integrals in the complex plane, and know and can use their basic properties and bounds.

(6) Cauchy's integral theorem and formula: you can state and prove Cauchy's integral theorem, apply it to prove Cauchy's integral formula, and use both to evaluate complex integrals. (CORE)

(7) Consequences of Cauchy's theorem and formula: you can apply these results to prove theorems such as Liouville's theorem, Morera's theorem, and the fundamental theorem of algebra.

(8) Power series and Taylor expansions, analytic continuation: you can express analytic functions on disks (including "at infinity") as power series, compute their radii of convergence, and understand how analytic continuation can be used to define certain analytic functions outside their radius of convergence.

(9) Laurent series and singularities: you can express analytic functions on annuli as Laurent series, classify their isolated singularities and singularities "at infinity," and describe the behavior of analytic functions near each type of singularity.

(10) Residue theorem and applications: you can compute residues at isolated singularities and use the residue theorem to evaluate contour integrals, including real integrals and sums, as well as integrands with branch points. (CORE)

(11) The argument principle and Rouché's theorem: you can state, prove, and apply the argument principle to count zeros and poles of meromorphic functions inside a contour, deduce Rouché's theorem from the argument principle, and use it to study zeros of analytic functions in given regions.

(12) The Schwarz lemma and (some) hyperbolic geometry: you understand and can apply the Schwarz lemma for conformal self-maps of the unit disk, and are familiar with some applications to the hyperbolic metric.

(13) Poisson integration formula and the characterization of harmonic functions: you understand and can apply the Poisson integral formula, and can characterize harmonic functions in terms of the mean value property.

(14) Conformal mappings and the Riemann mapping theorem: you can construct explicit conformal maps between standard domains, can state the Riemann mapping theorem, and can apply it to classify simply connected domains in the Riemann sphere.

It is possible, though hopefully unlikely, that we will not get to some of the final topics due to time constraints. If so, the objective requirements will be adjusted suitably: e.g. if we need to drop an objective, only 13 will be required for an A+, 12 for an A, etc.

Course policies

Attendance

Attendance is not mandatory, in the sense that I will not be taking attendance and it is not part of your grade, but it is expected as a part of the class. Empirically, students tend to do better when they come to class. However if you need to miss a class for any reason, **you**

do not need to inform me.

If you need to miss a midterm, contact me **beforehand**, or as soon as possible afterwards in case of illness, and we can try to work something out.

Deadlines and extensions

If you are unable to turn in your homework by the official deadline, please contact me **at least 24 hours in advance** with a request for an extension; you must specify when you would like the new deadline to be. I expect to grant all such requests, within reason, but try not to do this more than you have to.

Late homework may be marked late, potentially more than a week even after it is submitted, since the grader will need to prioritize new work. In particular, it may not be graded in time to unlock challenge problems on the next midterm or retest, for which there is no recourse. Late homework submitted without an extension may not be marked at all.

Technology requirements

You do not need any particular technological devices for this class other than a writing implement (for exams, at least); in particular you do not need a calculator. However, many materials will by default only be made available online, via bCourses, and homework submissions will also be online. If this presents a difficulty for you contact me and I'll make the materials available on paper as needed.

COVID-19 policies

Classes will be in person, but please do not come to class if you are feeling sick or test positive: lecture notes will be available, and I will be happy to help you make up the material. If you are sick and unable to do the work for a prolonged period, contact me to work out a way to make up the work: it is important to do the work for each section of the class, since that is the way to learn the material and later portions of the class will build on the earlier ones, but when necessary we can figure out how to reduce the workload to be manageable without having to work through illness. Similarly, please do not attend exams if you are ill or have recently tested positive for COVID-19: we will figure out solutions if these situations arise.

Accessibility and accommodations

Please let me know if there is anything I can do to make this course more accessible to you, or if aspects of the course are excluding you, and we can work together to develop strategies to improve the class. If you think you may need official accommodations, such as extended time on exams, I encourage you to contact the Disabled Students' Program for an accommodation letter.

Tentative course outline

As promised:

Aug. 27, 29	Complex numbers and geometry
Sept. 3, 5	Some elementary functions, review of analysis
Sept. 8, 10, 12	Analytic functions
Sept. 15, 17, 19	More analytic functions, exam 1
Sept. 22, 24, 26	Line integrals and harmonic functions
Sept. 29, Oct. 1, 3	Complex integration, Cauchy's integral theorem and formula
Oct. 6, 8, 10	Applications of Cauchy's formula
Oct. 13, 15, 17	Exam 2, power series
Oct. 20, 22, 24	Series expansions of analytic functions
Oct. 27, 29, 31	Analytic continuation, Laurent series
Nov. 3, 5, 7	Singularities and the residue theorem
Nov. 10, 12, 14	Exam 3, the argument principle and Rouché's theorem
Nov. 17, 19, 21	The Schwarz lemma, the Poisson integral formula
Nov. 24	Characterization of harmonic functions
Dec. 1, 3, 5	Conformal mappings, review
Dec. 8, 10, 12	Review week