Exercise 1. For each of the following functions, find the domain and the range. Determine whether the function is 1-1 and, in the affirmative case, find the inverse.

(a) \( f(x) = \frac{1}{1 - \tan x} \).

(b) \( f(x) = \sqrt{3 - x} + x \).

(c) \( f(x) = \sqrt{4 - x^2} \).

(d) \( f(x) = \frac{1}{\sqrt{x - 1}} \).

Exercise 2. (Trigonometric functions) Prove the following trigonometric identities.

(a) \( 1 + (\tan \theta)^2 = (\sec \theta)^2 \).

(b) \( 1 + (\cot \theta)^2 = (\csc \theta)^2 \).

(c) \( \tan(\frac{\pi}{2} - \theta) = \cot \theta \).

(d) \( \csc(\frac{\pi}{2} - \theta) = \sec \theta \).

(e) \( \sin(\arccos x) = \sqrt{1 - x^2} \).
    (Hint: denote \( \theta = \arccos x \), and use the relation \((\cos \theta)^2 + (\sin \theta)^2 = 1\).)

(f) \( \arcsin x + \arccos x = \frac{\pi}{2} \).

Exercise 3. (Hyperbolic functions) Consider the functions

\[
\cosh x = \frac{e^x + e^{-x}}{2}, \quad \sinh x = \frac{e^x - e^{-x}}{2},
\]
called hyperbolic cosine and hyperbolic sine.

(a) Prove that \( \sinh x = \frac{e^{2x} - 1}{2e^x} \) and \( \cosh x = \frac{e^{2x} + 1}{2e^x} \).

(b) Prove that \( (\cosh x)^2 - (\sinh x)^2 = 1 \).

(c) Prove that \( \cosh x \) is even and \( \sinh x \) is odd. Write \( e^x \) and \( e^{-x} \) as a sum of an even and an odd function.

(d) Prove that \( \sinh x \) is 1-1, find the range and the inverse function, denoted by \( \text{arsinh} \).

(e) Find the range of \( \cosh x \), and verify that it is not 1-1.