Answer key to Exercise Sheet 2

Functions II

Exercise 1. (a) The domain is $\mathbb{R} \setminus \left(\left\{\frac{\pi}{2} + n\pi \mid n \in \mathbb{Z}\right\} \cup \left\{\frac{\pi}{4} + n\pi \mid n \in \mathbb{Z}\right\}\right)$, the range is $\mathbb{R} \setminus \{0\}$, it is not 1-1.

(b) The domain is $(-\infty, 3]$, the range is $(-\infty, \frac{13}{4}]$, it is not 1-1.

(c) The domain is $[-2, 2]$, the range is $[0, 2]$, it is not 1-1.

(d) The domain is $[0, +\infty) \setminus \{1\}$, the range is $(-\infty, -1] \cup (0, +\infty)$, it is 1-1 and the inverse is $f^{-1}(x) = \left(\frac{1+x}{x}\right)^2$, from $(-\infty, -1] \cup (0, +\infty)$ to $\mathbb{R}$.

Exercise 2. (Trigonometric functions)
(a),(b),(e) Use the identity $(\cos \theta)^2 + (\sin \theta)^2 = 1$.
(c),(d),(f) Use the identities $\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$, $\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$.

Exercise 3. (Hyperbolic functions)
(a) Multiply by $e^x$.
(b) Compute.
(c) $e^x = \cosh x + \sinh x$, $e^{-x} = \cosh x - \sinh x$.
(d) The range is $\mathbb{R}$, the inverse function is $\text{arsinh } x = \ln\left(x + \sqrt{1+x^2}\right)$.
(e) The range is $[1, +\infty)$. It is not 1-1 because it is even, $\cosh 1 = \cosh(-1)$. 